General Equilibrium Model of Individual and Fund Investment

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Abstract

This paper analyzes static general equilibrium model with heterogenous individual investors who can either invest in homogenous funds or trade on their own account. Individuals differ in their abilities to process information about private signals and choose allocation of cash based on expected return of fund's and individual investment. I show that in equilibrium agents either invest on their own or invest in a fund, derive equilibrium price function and find that participation in individual investment depends on the value of public signal and price. The uniqueness of equilibrium is stated under additional conditions.

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1. Introduction

Asymmetries in information is one of the key factors for existence of trading on financial markets. As Grossman and Stiglitz (1980) show, there can be no trade if all the agents share the same information, risk aversion and endowments. On the other hand, asymmetries appear not only between individual agents but also between individuals and funds. Higher capacity of the latter provides them with a better position in markets and allows to derive higher returns. Thus individuals face double choice: investing on their own or in a fund and how much to learn.

In this paper I study a simple version of individuals-funds model in which individual agents can invest in financial markets both directly and indirectly through funds. In the former case they can collect private signals about final payoff of any stock while in the latter they delegate management of part of their money to a fund and invest the rest in a riskless asset. The question studied is what determines individual trading and under what conditions individuals are more likely to trade on own accounts. The paper also studies the consequences of information acquisition and delegation in equilibrium.

Individual investors along with funds have limited ability to process information. They may have scarce cognitive resources, limited time or insufficient education to investigate information about stocks effectively. Another bias individuals may face is their prior experience in some industry or firm which they know better and thus can learn about faster. These constraints induces the agents to move the allocation of time to stocks the agent can investigate better.

There are several approaches to constraints an agent faces. First thread of literature considers monetary costs of acquiring information. For example, Cespa (2008) studies information sales by a monopolist. Another body of literature considers entropy-based learning problems in which the capacity of agent's ability is limited. Namely, Peng and Xiong (2006) develop the model in which representative overconfident agents have limited time and divide it among learning stocks, sector factors or marker factor. They show that equilibrium is unique and the agent learns more about sectors and market than about the stock effects. Han (2008a, 2008b) studies optimal learning behavior under capacity constraints. He derives the closed-form solution for agents' information acquisition problem and shows the uniqueness of market equilibrium.

On the other hand, there is a number of papers studying delegation of management of wealth from individuals to funds. Garsia and Vanden (2005) study a model in which informed agents can establish mutual funds and thus indirectly sell information to uninformed households. They explain the size of funds and some equilibrium features, in particular, more informative asset prices and lower risk premium in presence of funds. Berk and Green (2004) show that the respond to past performance by individual investors can be reproduced in partial equilibrium model with heterogenous managers diffiring in abilities to capture "alpha". Cuoco and Kaniel (2006) develop a model of delegated asset management in the presence of performance-linked fees. There are three groups of agents: active investors, fund investors who invest in funds and fund managers. It is shown that several empirical facts like overinvestment in stocks included in benchmark and higher prices for these stocks can be replicated.

Guiso and Jappelli (2006) and Calvet, Campbell and Sodini (2008) provide two extensive empirical reviews of individual investment and information acquisition. Guiso and Jappelli (2006) shows that higher investment in information is more likely when the agent is overconfident and this leads to lower Sharpe ratio, more frequent trading, less delegation and less diversified portfolio. Calvet, Campbell and Sodini (2008) studies the dynamics for individual portfolios in Sweden. They show that wealthy and educated investors tend to rebalance their portfolios more actively, trade directly and tend to sell their individual portfolios fully when the portfolio behaved well recently. About a third of population invest on own accounts.

French (2008) and Fama and French (2008) provide detailed description of costs and benefits of active investing. They show that mutual funds are very close in their abilities to each other and that there is almost no persistence in returns. Moreover, it is shown that high cost of active management makes return even lower than one of the passive (market) portfolio.

Two papers are the most close to this one. Nieuwerburgh and Veldkamp (2008a, 2008b) develop a model in which agents can acquire information and differ in information acquisition capacity. They show that small initial differences in beliefs and ability to increase the precision of a private signal as convex function of capacity lead to under-diversification both of information acquisition and portfolio holdings. This approach helps to solve the home bias puzzle and the foreign investment puzzle.

In this paper, I model endogenous information acquisition in a multi-asset endowment economy and concentrate mainly on the choice of individual investors between proprietary trading and fund investment. First, I study the choice between funds and own account and show that low abilities prevent agents to invest directly even in "good times" when prices and expected payoff are high and that agents choose either fund or own account for investment. Second, I show that exogenous differences in abilities to learn information can result in poorly diversified portfolios of individual agents who invest in stocks directly. Third, I derive equilibrium prices and show that the optimal behavior of better educated agents is similar to one of the agents with low abilities: namely, increase in belief about final payoff or prices leads to lower direct investment. The share of agents who invest in funds is endogenous and derived in equilibrium.

The paper is structured as follows. Section 2 provides the overall framework of the model. In Section 3 I describe the optimal decision structure of individuals and funds. In Section 4 I solve the general equilibrium model with the continuum of identical funds and discuss some properties of the equilibrium. In Section 5 the summary of the main results is presented.

2. The Model

2.1. Assets

I consider a one-period pure exchange economy in which individual agents decide either to trade through mutual funds, to invest on their own or to buy riskless bond only, and acquire information if they decide to participate in trading. Funds acquire information on the payoff of the securities. After that, in the beginning of the period individuals and funds trade securities and consume the only good at the end of the period when payoffs are realized.

There are $N+1$ securities in the economy: riskless bond and N risky securities which are denoted by $j = 1, ..., N$. The bond is in a perfectly elastic supply and pays R units at the end of the period. The price of riskless asset is set to be one. The risky asset j pays $Z_j \sim N(\overline{Z}_j, (\sigma_{ii}^Z)^2)$ units at the end of the period where $Z_j, j = 1, ..., N$ are independent, exists in random supply $S_j \sim N(S_{j0}, \nu^2)$ and its price is P_j. Denote $Z = (Z_1, ..., Z_N)^T$, $S = (S_1, ..., S_N)^T$, $S_0 = (S_{10}, ..., S_{N0})^T$ and $P = (P_1, ..., P_N)^T$. I assume that $S_j, j = 1, ..., N$ are independent and $\widetilde{S} = S - S_0$ is independent of Z and all other error terms.

2.2. Agents: individuals

There is a continuum of agents indexed by $a \in A = [0, 1]$. The measure on this interval is uniform. The agents have no impact on prices.

There are two types of agents in the model: funds indexed by $b \in [0, a_0]$ which have no initial capital and individuals indexed by $a \in (a_0, 1]$ who possess cash. I assume that agent $a \in (a_0, 1]$ has cash w_{0a} at the start of the period.

Individuals have two ways to invest. First, they can choose to trade on their own (so called "proprietary traders"). In this case they collect information on the realization of the payoff of risky assets and trade paying costs depending on the time spent on collecting information. Denote l_{ai} the time which agent a spends to investigate asset i. I assume that

$$l_a = l_{a1} + ... + l_{aN} \leq 1. \tag{1}$$

There is a uniform cost of trading for the individual which is equal to τ_0. This cost prevents the agents with low abilities to trade on their own accounts if the fee of the fund is low enough.

There may also be (financial) cost to spend time to collect information; I do not consider this cost in the current version since I concentrate on the choice between individual and fund investment, and not on the choice of time to learn.

Second option is to invest through a fund $b \in [0, a_0]$. In this case the individual bears no cost but pays a fraction c_b of the final wealth to the fund. I assume for the static model that the funds are homogenous in abilities and the agents choose them randomly, that is, in equilibrium each fund has the same wealth under management. This assumption is restrictive, yet it is a plausible first step to solve the delegation-trading problem. Despite the funds have the same abilities, they differ in their exposure to the market because of different private signals they receive.

The agent may combine both options and learn and trade for herself at the same time as she invests into the fund. I show in section 3.2 that the agents who trade are unwilling to invest in the fund. Intuitively, the agents who decide to trade can fully replicate the average position the fund holds and thus are reluctant to spend their money with the fund. However, any individual investor holds a so-called passive portfolio based on public news and prices, and may be considered as both fund and proprietary investor.

Individuals differ in their abilities to acquire and interpret the information. I discuss the learning process in section 2.4. For simplicity all the individuals are divided into the subgroups by their abilities and they are assumed to have the same risk aversion.

Denote by $\alpha_a = (\alpha_{a1}, ..., \alpha_{aN})^T, a \in [0, 1]$ the number of shares of respective risky assets bought by agent a. The final payoff of agent a investing on her own and investing amount $w_{0a}^f \geq 0$ in the fund b is then

$$W_{a1} = (w_{0a} - w_{0a}^f)R + \alpha_a^T(Z - PR) + (1 - c_b)w_{0a}^f \xi_{ab} - \tau_0 \chi_{trade}. \tag{2}$$

Here χ_{trade} is an indicator function equal to 1 if the agent trades and 0 otherwise. The definition of ξ_{ab} and its distribution will be stated fully in section 3.1 where the fund's problem is solved. This variable reflects the return the fund is paying to the agent.

To have a tractable equilibrium notion I assume that the parameters of the model and the distribution of price are known to any agent.

Agent a maximizes a constant absolute risk aversion (CARA) utility with absolute risk aversion coefficient $\rho_a = \rho_0$ less the cost of trading:

$$U(W_{0b}, l_b) = -\mathbf{E_0} \exp(-\rho_0 W_{1a}). \tag{3}$$

I denote by $\mathbf{E}_0$ and $\mathbf{V}_0$ the expectation and variance conditional on public information (that is, on price and public news) and by $\mathbf{E}_1$ and $\mathbf{V}_1$ the expectations conditional on public information and learning decisions.

2.3. Agents: funds and investment process

I assume that any fund $b \in [0, a_0]$ sets the same fee $c_b = c$, has the same risk aversion $\rho_b = \rho_1$, the same vector of learning abilities $\zeta_f = (\zeta_{f1}, ..., \zeta_{fN})$ and maximizes a constant absolute risk aversion (CARA) utility

$$U(W_{0b}, l_b) = -\mathbf{E_0} \exp(-\rho_f c W_{1b}). \tag{4}$$

All funds are assumed to have the same level of learning abilities (see next section). I also assume that the funds bear no trading cost.

The question which should be discussed is why the fund collects money from individual agents ? In the model funds may instead borrow any amount of cash under rate R and invest it to learned stocks privatizing all the profits. The answer may be one of two. First, we can assume that the funds can not borrow if they have no money under management. Second, we can also impose restriction that any fund can only invest the amount of money collected from individuals. Both of the answers are disputable because position held by the fund is big enough such that it may have to borrow from the spot market to invest into risky security. As usual for the models with Gaussian-linear framework (in which, say, prices may be negative) the assumption could be made that fund's investments are not so high so that the fund does not need to borrow from the market. This would make the fund a reasonable part of the model.

2.4. Information acquisition and learning process

There are three types of information in the model - public news, prices and private information. Public news and prices mainly attract people to invest in the stock market (individuals are less likely to invest when stock market goes down), while private signals allow to make better estimates of the payoff.

Agents can learn from prices, but this learning is not a full substitute of private information. Price is a noisy signal about the final payoff because supply of the assets is random and unobservable to the agents.

I assume that in the beginning of the period agents observe public signal $Y = Z + \epsilon_Y$, here ϵ_Y has multidimensional normal distribution with parameters $(0, \Sigma^Y)$ and Σ^Y is diagonal. Error term ϵ_Y is independent of Z and other errors.

The information in prices is interpreted rationally. In linear equilibrium they are the functions of the parameters of the model, public signal, payoffs and unknown supply:

$$P = \frac{1}{R}(A_1 + A_2 Y + A_3 Z - A_4 \widetilde{S}). \tag{5}$$

Matrices A_i are computed in Appendix 3. Note that A_1 is $N \times 1$ vector, A_2, A_3 and A_4 are $N \times N$ diagonal matrices which are assumed to be non-singular (this is proven in Appendix 3).

Knowing this function the agents can infer some information from prices and thus they know that, conditional on the information available after the signal collection, the following holds: $Z = A_3^{-1}(RP - A_1 - A_2 Y) + A_3^{-1} A_4 \widetilde{S}$. This information is used to update the beliefs. Denote by $\widetilde{P} = A_3^{-1}(RP - A_1 - A_2 Y)$ and by $\Sigma^p = (A_3^{-1} A_4)^T A_3^{-1} A_4 \nu^2$. Then the distribution of $\widetilde{P}$ conditional on Z is normal with the mean Z and the variance Σ^p.

Taking into account prices and public information the agents form the public belief on the final payoff according to the Bayes' law:

$$\widehat{Y}_{pub} = \left((\Sigma^Y)^{-1} + (\Sigma^Z)^{-1} + (\Sigma^p)^{-1}\right)^{-1} \left((\Sigma^Y)^{-1}Y + (\Sigma^Z)^{-1}\overline{Z} + (\Sigma^p)^{-1}\widetilde{P}\right), \tag{6}$$

$$\Sigma_{pub} = \left((\Sigma^Y)^{-1} + (\Sigma^Z)^{-1} + (\Sigma^p)^{-1}\right)^{-1}. \tag{7}$$

The agent can also collect a private signal on the realization of the payoff, $Y_{ai} = Z_i + \epsilon_{ai}$. Denote by $Y_a = (Y_{a1}, ..., Y_{aN})^T$. Private signals make information on the final payoff less uncertain: if the agent $a \in [0,1]$ applies effort l_{ai} to investigate asset i then the received signal Y_{ai} has the precision

$$\frac{1}{(\sigma_{ii}^{Y_a})^2} = \frac{\zeta_{ai} l_{ai}}{\left(\sigma_{ii}^{\widehat{Y}_{pub}}\right)^2}, \quad i = 1, ..., N. \tag{8}$$

Here $\sigma_{ii}^{Y_a}$ and $\sigma_{ii}^{\widehat{Y}_{pub}}$ are the diagonal elements of the matrices Σ^{Y_a}, Σ_{pub}.

In this setting ζ_{ai} is the expertise of agent a in asset i: the higher the ζ_{ai} the more precise signal would the agent get. To make things consistent, I allow the agents to have the level of expertise no greater than that of any fund, that is

$$\zeta_{ai} \leq \zeta_{bi}, \forall a \in (a_0, 1], b \in [0, a_0], i = 1, ..., N.$$

In the opposite case the agent may start her own fund which is prohibited in this model.

After collecting all the signals, the agents combine them using the Bayes' law and (8) to form the posterior mean and volatility conditional on signals:

$$\widehat{Y}_a^{learn} = \left((\Sigma^Y)^{-1} + (\Sigma^Z)^{-1} + (\Sigma^p)^{-1} + (\Sigma^{Y_a})^{-1}\right)^{-1} \times$$

$$\times \left((\Sigma^Y)^{-1}Y + (\Sigma^Z)^{-1}\overline{Z} + (\Sigma^p)^{-1}\widetilde{P} + (\Sigma^{Y_a})^{-1}Y_a\right), \tag{9}$$

$$\widehat{\Sigma}_a^{learn} = \left((\Sigma^Y)^{-1} + (\Sigma^Z)^{-1} + (\Sigma^p)^{-1} + (\Sigma^{Y_a})^{-1}\right)^{-1}. \tag{10}$$

The choice of exact form of the precision function in (8) makes the relation between $\widehat{\Sigma}_{pub}$ and Σ_a^{learn} very simple:

$$\frac{1}{\left(\sigma_{ii}^{\widehat{Y}_a^{learn}}\right)^2} = \frac{1 + \zeta_{ai} l_{ai}}{\left(\sigma_{ii}^{\widehat{Y}_{pub}}\right)^2}, \quad i = 1, ..., N. \tag{11}$$

Similar linear precision is mentioned in Han (2008) but there is an important difference: while in his paper the variance of additional signal is constant which does not depend on the variance of the combined public information, I assume that two variances are proportional. This makes the problem more tractable despite the loss of some features of the more general model, for example, dependance of the optimal time allocation on prior variances. At the same time, I preserve tractability of the solution and make it possible to express the choice of an individual agent in clear terms.

Note that all the matrices are assumed to be diagonal (otherwise it would be a non-trivial task to find a solution for equilibrium price function). This is not a restrictive assumption because payoffs and supplies of stocks are independent across stocks.

3. Investment decisions when funds are homogeneous

In this section I study the decisions of the agents to invest.

First, I derive the optimal allocation for the funds. The optimal solution for them is to learn about all the stocks (under the condition that the learning abilities are close to each other for all the stocks) and they spend equal time on each stock if the abilities are equal.

Second, I consider the choice of assets to learn by the individual investor knowing public information Y and prices. It is shown that in the equilibrium the agents either trade directly or delegate the trading choice to the funds. The intuition is straightforward: if the individual agent bears uniform cost of trading she can buy any portfolio and fully hedge herself from the fund's choice. Yet because the funds invest both in risky securities and bonds, it is better for the proprietary trader not to delegate. I show that the agent may choose one equity to investigate and holds "market" portfolio based on public news plus the learned security. The optimal decision to invest on their own for the agents with high enough learning abilities comes from the fact that the individual agent concentrates on one stock while the fund learns about all the stocks.

Finally, I derive the optimal fund holdings for the agents who do not invest into stocks directly.

The model is solved via backward induction.

3.1. Decision of the fund

Budget constraint for the fund b is given by

$$W_{1b} = w_{0b}R + \alpha_b^T(Z - PR). \tag{12}$$

The optimization problem is then

$$\max \mathbf{E}_0 \left[- \exp(-\rho_1 c(w_{0b}R + \alpha_b^T(Z - PR))) \right]. \tag{13}$$

Optimal portfolio can be written as

$$\alpha_b^* = \frac{1}{c\rho_1} \widehat{\Sigma}_{fund}^{-1}(\widehat{Y}^{fund} - PR), \tag{14}$$

In Appendix 1 it is shown that optimization problem reduces to minimizing the variance matrix under conditions (1) and (11):

$$(l_{b1}, ..., l_{bN}) = \arg\min \det(\widehat{\Sigma}_{fund}). \tag{15}$$

This means that CARA utility agents maximize the product of precisions $\prod\limits_{i=1}^{N} \frac{1}{(\widehat{\sigma}_{ii}^{fund})^2}$ because the matrix is diagonal. With linear learning function it results in the following equivalent maximization problem:

$$\max \sum_{i=1}^{N} \ln(1 + \zeta_{bi} l_{bi}) \quad s.t. \sum_{i=1}^{N} l_{bi} = 1,\ l_{bi} \geq 0. \tag{16}$$

First order conditions give equalities:

$$\frac{\zeta_{bi}}{\zeta_{bi} l_{bi} + 1} = \frac{\zeta_{bj}}{\zeta_{bj} l_{bj} + 1},\ \forall i, j : l_{bi} > 0, l_{bj} > 0. \tag{17}$$

If we assume that the fund b has the same expertise over all the assets it would mean that it invests the same amount of time in each.

The following inequality means that the quality of the learned signal, that is, its precision, is at least as high as one of public signal if the fund spends all the time to learn about one asset:

$$\zeta_{bi} > 1, \forall i, b. \tag{18}$$

Denote by (i) the rank of the number among $\zeta_{b1}, ..., \zeta_{bN}$: $\zeta_{b(1)} \leq \zeta_{b(2)} \leq ... \leq \zeta_{b(N)}$. That is, $\zeta_{b(1)}$ is the lowest number among $\zeta_{b1}, ..., \zeta_{bN}$ and $\zeta_{b(N)}$ is the highest one, while, say, $\zeta_{b(5)}$ is the fifth-low. If two numbers are equal then two near-ranked numbers are equal. Set also $\zeta_{b(0)} \equiv 0$.

Next proposition states the exact form of learning for the fund.

Proposition 1. Let $1 \leq m \leq N$ be the number such that

$$\frac{1}{\zeta_{b(m)}} < \frac{1}{N-m+1}\left(1 + \sum_{i=m+1}^{N} \frac{1}{\zeta_{b(i)}}\right), \quad \frac{1}{\zeta_{b(m-1)}} \geq \frac{1}{N-m+2}\left(1 + \sum_{i=m}^{N} \frac{1}{\zeta_{b(i)}}\right).$$

Then:

1) the fund investigates assets on which it has expertise $\zeta_{b(m)}, ..., \zeta_{b(N)}$;

2) the lowest number of assets the fund b learns about is 2, i.e. $m \leq N-1$;

3) the optimal time spent is

$$l_{b(i)} = \frac{1}{N-m+1}\left[1 + \sum_{j=m}^{N}\left(\frac{1}{\zeta_{b(j)}} - \frac{1}{\zeta_{b(i)}}\right)\right], \; i = m, ..., N.$$

The proof of this proposition is presented in Appendix 2.

Let's consider two examples.

1) If we assume that fund b has the same expertise over all the assets it would mean that it invests $\frac{1}{N}$ to learn about each. What differs for the fund is the position which the fund holds in risky asset - other things equal, the higher the ability the higher are the holdings in absolute terms.

2) If the fund has low ability in one stock (say $\zeta_{b1} < 2$) and high and equal in others (say $\zeta_{bi} = 4$) then it will not learn the first stock because it is too costly to investigate information, the ex ante utility will increase more if it learns other stocks.

Now we can characterize the exact form of ξ_{ab}. If the fund possesses the amount of cash w_{0b} in the beginning of the period then formulas (12) and (14) give the end-of-period wealth:

$$W_{1b} = w_{0b}R + \frac{1}{c\rho_1}(\widehat{Y}^{fund} - PR)^T \widehat{\Sigma}^{-1}_{fund}(Z - PR). \tag{19}$$

The fund pays to investors amount proportional to their investment to the fund and thus $\xi_{ab} = \frac{1}{w_{0b}} W_{1b} = R + \frac{1}{c\rho_1 w_{0b}}(\widehat{Y}^{fund} - PR)^T \widehat{\Sigma}^{-1}_{fund}(Z - PR)$.

3.2. Individual's problem: proprietary trading vs fund investment

Given (2) and the exact form of ξ_{ab} we can write down the time-1 wealth of agent a in the following form:

$$W_{1a} = w_{0a}R - w^f_{0a}cR + \left(\alpha^T_a + \frac{(1-c)w^f_{0a}}{c\rho_1 w_{0b}}(\widehat{Y}^{fund}_a - PR)^T \widehat{\Sigma}^{-1}_{fund}\right)(Z - PR) - \tau_0 \chi_{trade}.$$

Here $\widehat{Y}^{fund}_a = \mathbf{E}(\widehat{Y}^{fund} | Y, P, Y_a)$.

Consider the optimal choice of this agent. We assume that $w^f_{0a} \geq 0$ and hence the agent can not short the fund. On the other hand, if she trades then the optimal solution for α_a is

$$\alpha_a + \frac{1-c}{c\rho_1 w_{0b}} \widehat{\Sigma}^{-1}_{fund}(\widehat{Y}^{fund}_a - PR) = \frac{1}{c\rho_0}\left(\widehat{\Sigma}^{learn}_a\right)^{-1}(\widehat{Y}^{learn}_a - PR).$$

This means that the agent can effectively undo all the actions the fund does and thus there is no reason to invest in the fund because it costs the agent a net loss $-w^f_{0a}cR$. The result can be formulated as follows:

Proposition 2. Any agent a either invests on her own or invests in a fund.

Consider the agent a who does proprietary trading. In this case the optimal allocation of the funds is

$$\alpha_a = \frac{1}{\rho_0}\left(\widehat{\Sigma}^{learn}_a\right)^{-1}(\widehat{Y}^{learn}_a - PR). \tag{20}$$

Utility conditional on P and Y can be rewritten as (see appendix 1)

$$U(W_{1a}|Y,P) = -\left(\frac{\det(\Sigma_a^{learn})}{\det(\Sigma_{pub})}\right)^{1/2} \exp(-\rho_0(w_{0a}R-\tau_0)-0.5(\widehat{Y}_{pub}-PR)^T\Sigma_{pub}^{-1}(\widehat{Y}_{pub}-PR))).$$

As in the previous section the solution for the individual problem is the solution for the maximization problem:

$$\max \sum_{i=1}^{N} \ln(1+\zeta_{bi}l_{bi}) \quad s.t. \sum_{i=1}^{N} l_{bi} = 1,\, l_{bi} \geq 0.$$

The purpose is to make agents learn only one stock because this would create undiversified portfolio unlike the fund's one. To achieve this goal I assume that the abilities of the agent are the same except for one. Namely, for a given $1 \leq i_a \leq N$

$$\zeta_{ai} = \zeta_a,\, i,j = 1,...,N,\, i,j \neq i_a; \zeta_{ai_a} > \zeta_a.$$

Condition to make i_a the only stock which agent a will learn about can be derived using proposition 1.

Proposition 3. Agent a learns stock i_a if the following condition is satisfied:

$$\zeta_{i_a} \geq \frac{1}{\frac{1}{\zeta_a} - 1}. \tag{21}$$

The proof is presented in Appendix 2.

There is no assumption like (18) for the individuals and thus the only asset which is learned is i_a if $\zeta_a < 1$ and (21) holds. By means of (11) the utility function can be expressed as

$$U(W_{1a}|Y,P) =$$

$$= -(1+\zeta_{i_a})^{1/2} \exp(-\rho_0(w_{0a}R-\tau_0) - 0.5(\widehat{Y}_{pub} - PR)^T\Sigma_{pub}^{-1}(\widehat{Y}_{pub} - PR))). \tag{22}$$

3.3. Individual investment: fund

In this section we solve the agent's problem if she invests into the fund. All the funds have the same abilities and the agents choose them randomly. By the law of large number for continuum of random variables the share of individuals who invest in a fund is the same for all funds.

Wealth at time 1 for this agent is given by

$$W_{1a} = w_{0a}R - w_{0a}^f cR + \frac{(1-c)w_{0a}^f}{c\rho_1 w_{0b}}(\widehat{Y}_a^{fund} - PR)^T \widehat{\Sigma}_{fund}^{-1}(Z - PR). \tag{23}$$

Here $\widehat{Y}_a^{fund} = \mathbf{E}(\widehat{Y}^{fund}|Y,P) = \widehat{Y}_{pub}$ because the agent only knows public information about the payoff. At the same time, since the fund investigates Z, agent a has a prior $Z \sim N(\widehat{Y}_{pub}, \Sigma_{fund})$. There is no learning problem for the agent a and hence she solves the mean-variance maximization problem taking w_{0b}, P and Y as given:

$$\mathbf{E}W_{1a} - \frac{\rho_0}{2}\mathbf{V}W_{1a} = -w_{0a}^f cR + \frac{(1-c)w_{0a}^f}{c\rho_1 w_{0b}}(\widehat{Y}_{pub} - PR)^T \widehat{\Sigma}_{fund}^{-1}(\widehat{Y}_{pub} - PR) -$$

$$-\frac{\rho_0}{2}\frac{(1-c)^2(w_{0a}^f)^2}{c^2\rho_1^2(w_{0b})^2}(\widehat{Y}_{pub} - PR)^T \widehat{\Sigma}_{fund}^{-1}(\widehat{Y}_{pub} - PR) \to \max \tag{24}$$

Denote by $t = (\widehat{Y}_{pub} - PR)^T \widehat{\Sigma}_{fund}^{-1}(\widehat{Y}_{pub} - PR)$. First order condition for the problem is:

$$-cR + \frac{(1-c)t}{c\rho_1 w_{0b}} - \frac{\rho_0(1-c)^2 w_{0a}^f t}{c^2\rho_1^2(w_{0b})^2} = 0,$$

$$w_{0a}^f = \frac{c^2\rho_1^2}{\rho_0(1-c)^2 t}\left(-cR(w_{0b})^2 + \frac{(1-c)tw_{0b}}{c\rho_1}\right). \tag{25}$$

The first conclusion is that we have to solve the model in general equilibrium framework because otherwise w_{0b} can not be correctly defined. Second, the optimal supply is a quadratic function of the fund's size which is intuitive: if the fund is big enough it will invest most of its funds into riskless asset (the feature of CARA utility) and the return will be low. In this case the agents prefer to invest on their own.

4. General equilibrium

In this section I consider general equilibrium results for optimal individual and fund investment. To make things more tractable I solve a special case in which all the individual agents are divided into subgroups by their learning abilities and in particular by the stock they investigate. I assume that the abilities of the funds are the same across stocks and thus each fund spends equal time on every asset. The share of agents investing in funds is assumed to be constant and the optimal size of the fund is derived.

4.1. The model: additional assumptions

To make the model simpler and at the same time to save its specific features I assume the following.

First, the share of agents who do not invest in the fund is set to be $1-\delta$, $0<\delta<1$. This means that the measure $\delta(1-a_0)$ of individuals does not invest in stocks directly and $(1-\delta)(1-a_0)$ trades on their accounts. I show that in the equilibrium setting for the one-period model δ is unique under additional assumptions on parameters.

Second, the agents who may invest in assets directly are divided into N subgroups of equal size $\frac{(1-\gamma)(1-a_0)}{N}$ and $\gamma \leq \delta$. Subgroup $i=1,...,N$ studies stock i. This assumption can be made if we use proposition 3 and assume that the abilities in group i are such that $\zeta_{ai}=\zeta_0>\frac{1}{\frac{1}{\zeta_{aj}}-1}$, $\forall j\neq i$. This group is called "agents with high abilities". Assume also that the rest $\gamma(1-a_0)$ of agents have the same structure as those who may invest on their own. That is, the abilities of these agents have one maximum at some $i=1,...,N$: $\zeta_{ai}=\zeta_2>\frac{1}{\frac{1}{\zeta_{aj}}-1}$, $\forall j\neq i$. This group will be called "agents with low abilities".

Finally, I assume that the abilities of any fund are the same across stocks, that is $\zeta_{bi}=\zeta_1$, $\forall b\in[0,a_0], i=1,...,N$. In this setting any fund will learn about all the stocks and divide time equally among them. This makes the solution especially tractable.

4.2. Optimal size of the fund and decision to invest in fund

Using the assumptions made in section 4.1 we can characterize the optimal size of the funds. The first condition is (25) which links the fund's size to individual investment into the fund. The second condition is based on macro level: the overall cash supplied to the funds should be equal to the total amount of cash collected by the funds, or

$$\delta(1-a_0)w_{0a}^{fund} = a_0 w_{0b}. \tag{26}$$

Combining (25) and (26) it can be shown that

$$w_{0b} = \frac{t(1-c)^2\rho_0}{Rc^3\rho_1^2}\left(\frac{\rho_1 c}{\rho_0(1-c)} - \frac{a_0}{\delta(1-a_0)}\right), \tag{27}$$

$$w_{0a}^{fund} = \frac{t(1-c)^2\rho_0 a_0}{(1-a_0)\delta Rc^3\rho_1^2}\left(\frac{\rho_1 c}{\rho_0(1-c)} - \frac{a_0}{\delta(1-a_0)}\right). \tag{28}$$

Note that the only parameter which is endogenous in these expressions is δ and in the next section the decision of an individual shows that unique δ can be supported in equilibrium.

Both w_{0b} and w_{0a}^{fund} can be negative (in this case they should be 0). Consider, however, reasonable parameters for US economy. In 2006 there were approximately 8000 mutual funds which serve 100 millions US citizens. It means that $a_0/(1-a_0) \approx 10^{-4}$. There is no good proxy for δ in the US but the evidence in Calvet et al. (2007) shows that for Sweden $\delta = 0.7$ (and probably higher for the US because the Sweden population is extensively educated in financial decisions). Average fees for mutual funds are about 0.5% and thus $c/(1-c) \approx 5 \times 10^{-3}$. Hence, if the risk aversion coefficients of funds and individuals are not 20 times different, the expression in brackets in (27) is positive. Given all this information I assume that

$$\frac{\rho_1 c}{\rho_0(1-c)} > \frac{a_0}{\gamma(1-a_0)}. \tag{29}$$

The agent can decide either to invest in a fund or on his own even if she has high abilities. The optimal condition is now stated. The utility from investing on own accounts given P and Y for the agent with maximum learning ability ζ is like in (22):

$$U_1(W_{1a}|Y,P) =$$

$$= -(1+\zeta)^{1/2}\exp(-\rho_0(w_{0a}R-\tau_0) - 0.5(\widehat{Y}_{pub}-PR)^T\Sigma_{pub}^{-1}(\widehat{Y}_{pub}-PR))).$$

To derive utility if the agent invests in fund we use (23) and (27-28):

$$\mathbf{E}W_{1a} - \frac{\rho_0}{2}\mathbf{V}W_{1a} = w_{0a}R + \frac{1}{2}\frac{a_0^2\rho_0(1-c)^2}{\delta^2(1-a_0)^2\rho_1^2c^2}t.$$

Thus the utility is:

$$U_2(W_{1a}|Y,P) = -\exp\left[-\rho_0\left(w_{0a}R + \frac{1}{2}\frac{a_0^2\rho_0(1-c)^2}{\delta^2(1-a_0)^2\rho_1^2c^2}t\right)\right].$$

Recall that the fund investigates all the stocks with the same amount of time and we can write:

$$t = (\widehat{Y}_{pub}-PR)^T\widehat{\Sigma}_{fund}^{-1}(\widehat{Y}_{pub}-PR) = (1+\zeta_1/N)(\widehat{Y}_{pub}-PR)^T\Sigma_{pub}^{-1}(\widehat{Y}_{pub}-PR).$$

Denote by $t_{pub} = (\widehat{Y}_{pub}-PR)^T\Sigma_{pub}^{-1}(\widehat{Y}_{pub}-PR)$. The agent chooses to invest on her own iff $U_1 > U_2$ or

$$0.5\log(1+\zeta) + \rho_0(w_{0a}R-\tau_0) + 0.5t_{pub} > \rho_0\left(w_{0a}R + \frac{1}{2}\frac{a_0^2\rho_0(1-c)^2}{\delta^2(1-a_0)^2\rho_1^2c^2}(1+\zeta_1/N)t_{pub}\right).$$

The last inequality can be rewritten as:

$$\log(1+\zeta) > 2\rho_0\tau_0 + t_{pub}\left(\frac{a_0^2\rho_0^2(1-c)^2}{\delta^2(1-a_0)^2\rho_1^2c^2}(1+\zeta_1/N) - 1\right). \tag{30}$$

Assume that:

$$\frac{a_0^2\rho_0^2(1-c)^2}{(1-a_0)^2\rho_1^2c^2}(1+\zeta_1/N) > 1, \tag{31}$$

$$\log(1+\zeta_2) \le 2\rho_0\tau_0. \tag{32}$$

Given (31-32) the agents with low abilities will always invest in a fund for any realization of Y, P.

Inequality (30) also provides the bound for ζ_0: if $\log(1+\zeta_0)$ is higher than the right part of the inequality the agent should invest on her own. If this is not the case then in the equilibrium $\delta > \gamma$, that is, only a part of individuals with high abilities will trade directly. There are three cases which can be realized.

1) All individuals with high abilities invest directly.

This case takes place if:

$$\log(1+\zeta_0) > 2\rho_0\tau_0 + t_{pub}\left(\frac{a_0^2\rho_0^2(1-c)^2}{\gamma^2(1-a_0)^2\rho_1^2c^2}(1+\zeta_1/N)-1\right). \tag{33}$$

Given this inequality all the agents with high abilities are proprietary traders because their expected utility from individual investment is higher.

2) A fraction $0 < 1-\delta < 1-\gamma$ of individuals invest directly. We assume that in each group of agents with high abilities some $\frac{\delta-\gamma}{1-\gamma}$ invest in funds and $\frac{1-\delta}{1-\gamma}$ invest on own accounts. Then the total mass of agents who invest in funds is $\gamma + (1-\gamma)\frac{\delta-\gamma}{1-\gamma} = \delta$.

The equality for this case is:

$$\log(1+\zeta_0) = 2\rho_0\tau_0 + t_{pub}\left(\frac{a_0^2\rho_0^2(1-c)^2}{\delta^2(1-a_0)^2\rho_1^2c^2}(1+\zeta_1/N)-1\right). \tag{34}$$

The equality means that the agents with high abilities are indifferent between investing in a fund and on their own. This is a Nash equilibrium because nobody is better off playing another strategy.

3) All the agents invest in a fund.

This case happens when

$$\log(1+\zeta_0) < 2\rho_0\tau_0 + t_{pub}\left(\frac{a_0^2\rho_0^2(1-c)^2}{(1-a_0)^2\rho_1^2c^2}(1+\zeta_1/N)-1\right). \tag{35}$$

The intuition is that for high realization of t_{pub} individuals expect the funds to do a better job and derive higher returns from the investment. There is no reason not to delegate.

Note that right side of inequalities (33-35) is a continuous function of δ and thus the equilibrium exists for any parameters. Interesting question is under what conditions this equilibrium is unique. If inequalities 33 or 35 hold then the equilibrium is the one described: either all individual agents with high abilities trade (under (33)) or nobody trades (under (35)).

In Appendix 3 I derive the price function. I show that coefficients at the right side of equations (33-35) decrease in δ under some technical conditions and the equilibrium is unique. I put an important condition here:

$$\zeta_0 \geq \zeta_1 - \frac{a_0\rho_0(1-\gamma)\zeta_1(1+\zeta_1/N)}{c\rho_1}. \tag{36}$$

This condition represents the idea that individuals have high enough abilities to process the information. If it is not satisfied there may be a multiplicity of equilibriums satisfying (34).

4.3. Analysis of equilibrium

The question is what would be the response of δ, P and return to portfolio to changes in parameters and/or realizations of random variables of the model ?

To answer this question I consider the unique equilibrium derived in the previous section. Denote $f(\delta) = t_{pub}\left(\frac{a_0^2\rho_0^2(1-c)^2}{\delta^2(1-a_0)^2\rho_1^2c^2}(1+\zeta_1/N)-1\right)$. In appendix 3 it is shown that this function is decreasing in δ.

First, since f is decreasing in δ the optimal δ increases with increase in Y, Z and decrease in $\widetilde{S}$. The result follows from the fact that t_{pub} shifts up with this changes in variables and higher δ is needed to cross the threshold $th = \log(1+\zeta_0) - 2\rho_0\tau_0$. This means that the share of individual agents investing on their own is decreasing with Y and $PR - A_2Y$.

Second, consider exposure to the market agents have. When an agent with high abilities trades on her own she buys an amount of risky asset stated in (20):

$$\alpha_a = \frac{1}{\rho_0}\left(\widehat{\Sigma}_a^{learn}\right)^{-1}(\widehat{Y}_a^{learn} - PR),$$

and her exposure to the market is

$$\alpha_a^T(\widehat{Y}_a^{learn} - PR) = \frac{1}{\rho_0}(\widehat{Y}_a^{learn} - PR)^T\left(\widehat{\Sigma}_a^{learn}\right)^{-1}(\widehat{Y}_a^{learn} - PR)$$

On average and given Z, Y, P this amount is equal to

$$\mathbf{E}_0\alpha_a^T(\widehat{Y}_a^{learn} - PR) = \frac{1}{\rho_0}\left(\Sigma_{pub}^{-1}(Y_{pub} - PR) + \Sigma_{pub}^{-1} \times diag(0, ..., \zeta_0, 0, ...)(Z - PR)\right)\Sigma_{pub}^{-1}\times$$

$$\times diag(1, ..., 1 + \zeta_0, 1, ...) \left(\Sigma_{pub}^{-1}(Y_{pub} - PR) + \Sigma_{pub}^{-1} \times diag(0, ..., \zeta_0, 0, ...)(Z - PR) \right).$$

If the agent invests in a fund (we abstract from the fact that this may not be possible for given Z, Y, P) then the amount invested is given by (28):

$$w_{0a}^{fund} = \frac{(1 + \zeta_1/N)(1 - c)^2 \rho_0 a_0 t_{pub}}{(1 - a_0)\delta R c^3 \rho_1^2} \left(\frac{\rho_1 c}{\rho_0(1 - c)} - \frac{a_0}{\delta(1 - a_0)} \right).$$

We can not compare the flows directly but can conclude that in the case of individual investment the holdings of risky securities are increasing in Y, P and Z while in the case of fund investment the amount of cash invested in fund is decreasing in δ because $\frac{1}{\delta^2} t_{pub}$ decreases in δ.

The portfolio individual agent holds is very undiversified: while one of the parts of this portfolio is a proxy for market portfolio, $\frac{1}{\rho_0}\Sigma_{pub}^{-1}(Y_{pub} - PR)$, the other part consists of one stock. This means that the agents are concentrated on the assets they know best which is consistent with empirical research. On the other hand, funds hold highly diversified portfolio and spend time on many stocks.

5. Conclusion

In this paper the model of individual and fund investment in a general equilibrium framework with information acquisition is developed. I show that the agents may hold undiversified portfolio with the stock they know most about and that funds investigate more stocks because of higher capacity. The unique equilibrium is derived under additional conditions reflecting high abilities of individual agents.

There are several features which can be included in the current version of the model. First, the funds may be heterogenous and in this case individuals may choose to invest in special fund learning only one industry because the expected returns in this industry are high for the individual agent. Funds can be heterogenous in fees or even set fees depending on the stocks they invest in.

Second, it is interesting whereas more challenging to study a model in which private signals have constant variance independent of the variance of the public signal. In this case linear equilibrium may be proven to be unique with no additional constraints but, at the same time, this makes it harder to solve individual problem.

The optimal choice of time spent to learn the stocks is another question. Assume that time is not free and agents bear some cost trying to acquire information. What would be the best choice for individual agent ? It is known that overconfident agents trade more and learn more while receive consistently lower return. The issue is whether this feature can be reproduced in this model or not.

The most interesting development is to consider a dynamic model in which the agents may collect information and trade for several periods before the final payoff realizes. This may make the model more sound because it reflects the ability of agents to increase their knowledge in time. Moreover, this could help to resolve the problems with the uniqueness of equilibrium. A dynamic model may also include performance-based behavior of individual agents like in Sirri, Tufano (1998): funds which perform better in the previous periods attract more finance than others. This type of behavior may be included even if the payoff is realized in the final period only because the funds generate returns in every period of the trading.

Appendix 1.

In this appendix I express utility of either an individual agent who invests on her own or a fund conditional on public information in terms of determinant of the variance-covariance matrix.

We use the formula in Brunnermeier (2001, p.64) for $z \sim N(0, \widetilde{\Sigma})$ and $I - 2\widetilde{\Sigma}F$ positive definite:

$$\mathbf{E}\left[\exp(z^T F z + G^T z + H)\right] = |I - 2\widetilde{\Sigma}F|^{-1/2} \exp\left(\frac{1}{2} G^T (I - 2\widetilde{\Sigma}F)^{-1}\widetilde{\Sigma}G + H\right).$$

Let's derive the formula for the fund, the expression for the individual is almost the same up to the change of the matrix, risk aversion and belief. We can substitute the optimal portfolio into the utility function expression:

$$\alpha_b^* = \frac{1}{c\rho_1}\widehat{\Sigma}_{fund}^{-1}(\widehat{Y}^{fund} - PR), \qquad (A1.1)$$

and hence the utility can be written as

$$U_{fund}(W_{1b}|Y,P) = \mathbf{E}_0\left[-\exp(-\rho_1 c w_{0b} R - \rho_1 c \alpha_b^T (Z - PR))\right] =$$

$$= \mathbf{E}_0\left[-\exp(-\rho_1 c w_{0b} R - 0.5(\widehat{Y}^{fund} - PR)^T \widehat{\Sigma}_{fund}^{-1}(\widehat{Y}^{fund} - PR))\right] =$$

$$= -det(I + (\widehat{\Sigma}_{fund} - \Sigma_{pub})^{-1/2}\widehat{\Sigma}_{fund}^{-1}) \exp(-\rho_1 c w_{0b} R - 0.5(\widehat{Y}_{pub} - PR)^T \widehat{\Sigma}_{pub}^{-1}(\widehat{Y}_{pub} - PR)) =$$

$$= -\sqrt{\frac{det(\widehat{\Sigma}_{fund})}{det(\Sigma_{pub})}} \exp(-\rho_1 c w_{0b} R - 0.5(\widehat{Y}_{pub} - PR)^T \widehat{\Sigma}_{pub}^{-1}(\widehat{Y}_{pub} - PR)). \qquad (A1.2)$$

The maximization of expected utility given public information is equivalent to maximization of $det(\widehat{\Sigma}_{fund})$ given (1) and (11).

Individuals have similar form of utility: they bear cost of trading and thus

$$U_a(W_{1a}|Y,P) = -\sqrt{\frac{det(\widehat{\Sigma}_a^{learn})}{det(\Sigma_{pub})}} \exp(-\rho_0 w_{0b} R + \rho_0 \tau_0 - 0.5(\widehat{Y}_{pub} - PR)^T \widehat{\Sigma}_{pub}^{-1}(\widehat{Y}_{pub} - PR)). \qquad (A1.3)$$

Appendix 2.

In this appendix propositions 1 and 3 are proven.

Proof of proposition 1:

Maximization problem the fund b solves is (16)

$$\max \sum_{i=1}^{N} \ln(1 + \zeta_{bi} l_{bi}) \quad s.t. \sum_{i=1}^{N} l_{bi} = 1,\ l_{bi} \geq 0.$$

First order conditions give equalities:

$$\frac{\zeta_{bi}}{\zeta_{bi} l_{bi} + 1} = \frac{\zeta_{bj}}{\zeta_{bj} l_{bj} + 1},\ \forall i, j : l_{bi} > 0, l_{bj} > 0.$$

These equations can be rewritten as

$$l_{bi} + \frac{1}{\zeta_{bi}} = l_{bj} + \frac{1}{\zeta_{bj}},\ \forall i, j = 1, ..., N. \qquad (A2.1)$$

Summing (A2.1) over j and using (1) we can express the time spent on stock i as

$$l_{bi} = \frac{1}{N} \left[1 + \sum_{j=1}^{N} \left(\frac{1}{\zeta_{bj}} - \frac{1}{\zeta_{bi}} \right) \right]. \qquad (A2.2)$$

The formula shows that the optimal time have the same order with respect to the abilities: if $\zeta_{bi_1} \leq \zeta_{bi_2} \leq ... \leq \zeta_{bi_N}$, $i_1, ..., i_N = 1, ..., N, i_s \neq i_t$ then $l_{bi_1} \leq l_{bi_2} \leq ... \leq l_{bi_N}$.

Consider the condition for internal solution $l_{bi} > 0$. If this condition is satisfied for l_{bi_1} then it will be satisfied for all the l_{bi} and thus the optimal decision for the fund is to learn about all the stocks with the time given by (A2.2). If $l_{bi_1} \leq 0$ then the fund does not investigate stock i_1 and the solution is $l_{bi_1} = 0$. Condition for that is

$$\frac{1}{\zeta_{bi_1}} \geq \frac{1}{N-1} \left[1 + \sum_{j=1, j \neq i_1}^{N} \frac{1}{\zeta_{bj}} \right].$$

Yet in this case conditions (A2.1) are not satisfied for i_1 and the optimal solution is based on formulas

$$l_{bi} = \frac{1}{N-1} \left[1 + \sum_{j=1, j \neq i_1}^{N} \left(\frac{1}{\zeta_{bj}} - \frac{1}{\zeta_{bi}} \right) \right],\ 1 \leq i \leq N, i \neq i_1. \qquad (A2.3)$$

Repeating the argument we change the system until we get the number $i_m, m \leq N$ such that:

$$l_{bi_m} = \frac{1}{N-m+1}\left[1 + \sum_{j=1, j\neq i_1, i_2, \ldots, i_{m-1}}^{N} \left(\frac{1}{\zeta_{bj}} - \frac{1}{\zeta_{bi_m}}\right)\right] > 0.$$

Note that this inequality is equivalent to

$$\frac{1}{\zeta_{bi_m}} < \frac{1}{N-m}\left[1 + \sum_{j=1, j\neq i_1, i_2, \ldots, i_{m-1}, i_m}^{N} \frac{1}{\zeta_{bj}}\right]. \qquad (A2.4)$$

By the induction step we also know that:

$$l_{bi_{m-1}} = \frac{1}{N-m+2}\left[1 + \sum_{j=1, j\neq i_1, i_2, \ldots, i_{m-2}}^{N} \left(\frac{1}{\zeta_{bj}} - \frac{1}{\zeta_{bi_{m-1}}}\right)\right] \leq 0$$

or simply:

$$\frac{1}{\zeta_{bi_{m-1}}} \geq \frac{1}{N-m+1}\left[1 + \sum_{j=1, j\neq i_1, i_2, \ldots, i_{m-2}, i_{m-1}}^{N} \frac{1}{\zeta_{bj}}\right]. \qquad (A2.5)$$

The optimal time spent can be derived from the first order conditions:

$$l_{bi} = \frac{1}{N-m+1}\left[1 + \sum_{j=1, j\neq i_1, i_2, \ldots, i_{m-1}}^{N} \left(\frac{1}{\zeta_{bj}} - \frac{1}{\zeta_{bi}}\right)\right]. \qquad (A2.6)$$

Equations (A2.4) and (A2.5) prove part 1 of proposition 1, equation (A2.6) - part 3 of proposition 1.

To prove part 2 we use condition (18): assume that $m = N$, that is, only one stock is learned. Then the condition on the second highest expertise should be in line with (A2.5)

$$\frac{1}{\zeta_{b(N-1)}} \geq \left[1 + \frac{1}{\zeta_{b(N)}}\right].$$

However, this can not be true by assumption (18) because the left side of last inequality is less than 1. □

Proof of proposition 3:

We use formula (A2.2) taking into account that all but one abilities are equal:

$$l_{aj} = \frac{1}{N}\left[1 + \sum_{j=1}^{N}\left(\frac{1}{\zeta_{aj}} - \frac{1}{\zeta_{ai}}\right)\right] = \frac{1}{N}\left[1 + \frac{1}{\zeta_{ai_a}} - \frac{1}{\zeta_a}\right],\ j \neq i_a.$$

This expression is non-positive iff

$$1 + \frac{1}{\zeta_{ai_a}} - \frac{1}{\zeta_a} \leq 0, \quad \zeta_{ai_a} \geq \frac{1}{\frac{1}{\zeta_a} - 1}.$$

Therefore, if (21) is satisfied then the agent learns only one stock. □

Appendix 3.

In this appendix I derive the equilibrium prices and show that the right side in inequalities (33-35) is decreasing in δ; this would mean that the equilibrium is unique. Assume that the fraction $1-\delta \leq 1-\gamma$ of individuals invest in stocks directly. Denote the group of agents studying stock i by G_i. Demand function of the agent a learning stock i is

$$D_{ai} = \frac{1}{\rho_0}(\Sigma_{ai}^{learn})^{-1}(Y_{ai}^{learn} - PR) =$$

$$= \frac{1}{\rho_0}(\Sigma_{pub}^{-1}(\widehat{Y}_{pub} - PR) + \Sigma_{pub}^{-1} \times diag(0,0,...,\zeta_0,0,...)(Y_a - PR)). \tag{A3.1}$$

Matrix $\Sigma_i^{learn} = \Sigma_{pub}^{-1} \times diag(0,0,...,\zeta_0,0,...)$ is measurable w.r.t. public information (this will be shown below) and thus by the exact law of large numbers for a continuum of independent random variables used in Sums (2006) and Han (2008)

$$\int_{a\in G_i} (\Sigma_i^{learn})^{-1}(Y_a - PR)da = (\Sigma_i^{learn})^{-1}(Z - PR).$$

Each group $G_i, i = 1,2,...,N$ has measure $\frac{1-\delta}{N(1-\gamma)}$ and thus the total demand from individuals is equal to

$$D_0 = \frac{1-\delta}{\rho_0(1-\gamma)}\left[\Sigma_{pub}^{-1}(\widehat{Y}_{pub} - PR) + \frac{1}{N}\Sigma_{pub}^{-1} \times diag(\zeta_0,...,\zeta_0)(Z - PR)\right] =$$

$$= \frac{1-\delta}{\rho_0(1-\gamma)}\left[\Sigma_{pub}^{-1}(\widehat{Y}_{pub} - PR) + \frac{\zeta_0}{N}\Sigma_{pub}^{-1}(Z - PR)\right]. \tag{A3.2}$$

Demand of the fund b is equal to

$$D_b = \frac{1}{c\rho_1}\left[\Sigma_{pub}^{-1}(\widehat{Y}_{pub} - PR) + \Sigma_{pub}^{-1}diag\left(\frac{\zeta_1}{N}, \frac{\zeta_1}{N}, ..., \frac{\zeta_1}{N}\right)(Y_b - PR)\right]$$

and the total demand of the funds is

$$D_1 = \frac{a_0}{c\rho_1}\left[\Sigma_{pub}^{-1}(\widehat{Y}_{pub} - PR) + \frac{\zeta_1}{N}\Sigma_{pub}^{-1}(Z - PR)\right]. \qquad (A3.3)$$

Equalizing supply $S_0 + \widetilde{S}$ and demand $D_0 + D_1$ we get

$$S_0 + \widetilde{S} = \left(\frac{1-\delta}{\rho_0(1-\gamma)} + \frac{a_0}{c\rho_1}\right)\Sigma_{pub}^{-1}(\widehat{Y}_{pub} - PR) + \left(\frac{(1-\delta)\zeta_0}{\rho_0(1-\gamma)N} + \frac{a_0\zeta_1}{c\rho_1 N}\right)\Sigma_{pub}^{-1}(Z - PR). \qquad (A3.4)$$

Remember that from (6) and (7)

$$\widehat{Y}_{pub} = \Sigma_{pub}((\Sigma^Y)^{-1}Y + (\Sigma^Z)^{-1}\overline{Z} + (\Sigma^p)^{-1}A_3^{-1}(PR - A_1 - A_2Y),$$

$$\Sigma_{pub}^{-1} = (\Sigma^Y)^{-1} + (\Sigma^Z)^{-1} + (\Sigma^p)^{-1}, \quad \Sigma^p = (A_3^{-1}A_4)^T A_3^{-1}A_4\nu^2.$$

It is also assumed that $PR = A_1 + A_2Y + A_3Z - A_4\widetilde{S}$. Denote by $C_1 = \frac{1-\delta}{\rho_0(1-\gamma)} + \frac{a_0}{c\rho_1}$ and $C_2 = \frac{(1-\delta)\zeta_0}{\rho_0(1-\gamma)N} + \frac{a_0\zeta_1}{c\rho_1 N}$. As (A3.4) has to hold for any realizations of $(Y, Z, \widetilde{S})$ then

$$\Sigma_{pub}S_0 = C_1\Sigma_{pub}(\Sigma^Z)^{-1}\overline{Z} - C_1A_1 - C_2A_1, \quad 0 = C_1\Sigma_{pub}(\Sigma^Y)^{-1} - C_1A_2 - C_2A_2, \quad (A3.5)$$

$$\Sigma_{pub} = -C_1\Sigma_{pub}(\Sigma^p)^{-1}A_3^{-1}A_4 + C_1A_4 + C_2A_4, \quad 0 = C_1\Sigma_{pub}(\Sigma^p)^{-1} - C_1A_3 + C_2I - C_2A_3. \qquad (A3.6)$$

Here I is $N \times N$ identity matrix. We see that the solution is measurable with respect to public information because it involves only known variance-covariance matrices and δ which depends on public variables.

Now we can solve the system using the fact that all matrices in the equations are diagonal. Consider equations $(A3.6)$. They involve matrices A_3 and A_4 and hence can be solved independently of equations $(A3.5)$. For simplicity, denote $a_3 = A_{3,ii}, a_4 = A_{4,ii}$. We rewrite $(A3.6)$ element by element i.e. for a given $i = 1, ..., N$:

$$\Sigma_{pub,ii} = \frac{(C_1 + C_2)a_4}{1 + C_1a_3/a_4\nu^2} = \frac{(C_1 + C_2)a_3 - C_2}{C_1a_3^2/a_4^2\nu^2}. \qquad (A3.7)$$

In this expression we take into account that $\Sigma^p = A_3^{-2} A_4^2 \nu^2$. Now a_4 can be expressed from $(A3.7)$:

$$a_4 = \frac{C_1 C_2 a_3/\nu^2}{(C_1 + C_2)a_3 - C_2}. \quad (A3.8)$$

Using second equation in $(A3.6)$ it can be shown that:

$$0 = C_1 \frac{\nu^2}{C_1^2 C_2^2}((C_1 + C_2)a_3 - C_2)^2 -$$

$$-((C_1 + C_2)a_3 - C_2)\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2} + \frac{\nu^2}{C_1^2 C_2^2}((C_1 + C_2)a_3 - C_2)^2\right).$$

We get rid of the root $a_3 = C_2/(C_1 + C_2)$ because a_4 is indefinite in this case and solve

$$0 = \frac{C_1 \nu^2}{C_1^2 C_2^2}((C_1 + C_2)a_3 - C_2) - \left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2} + \frac{\nu^2}{C_1^2 C_2^2}((C_1 + C_2)a_3 - C_2)^2\right). \quad (A3.9)$$

This equation is quadratic in $x = (C_1 + C_2)a_3 - C_2$ and it can be solved if the additional condition is satisfied:

$$\frac{\nu^2}{4C_2^2} - \frac{1}{(\sigma_{ii}^Z)^2} - \frac{1}{(\sigma_{ii}^Y)^2} > 0. \quad (A3.10)$$

This condition essentially means that the variances of supply, final payoff and public signal are high enough and thus public belief based on these sources is volatile enough. To make sure that the condition is satisfied for any δ I set

$$\frac{\nu^2}{4\left(\frac{\zeta_0}{\rho_0 N} + \frac{a_0 \zeta_1}{c \rho_1 N}\right)^2} - \frac{1}{(\sigma_{ii}^Z)^2} - \frac{1}{(\sigma_{ii}^Y)^2} > 0, \forall i = 1, ..., N. \quad (A3.11)$$

This condition is easy to satisfy for high N and c, a_0 used in the text.

Deriving x from $(A3.9)$ we can get

$$x = \frac{C_1}{2} \pm \frac{\sqrt{\frac{\nu^4}{C_1^2 C_2^4} - 4\frac{\nu^2}{C_1^2 C_2^2}\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}}{2\nu^2/C_1^2 C_2^2}$$

and therefore:

$$a_3 = \frac{C1 + 2C_2}{2C_1 + 2C_2} \pm \frac{\sqrt{\frac{\nu^4}{C_1^2 C_2^4} - 4\frac{\nu^2}{C_1^2 C_2^2}\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}}{2(C_1 + C_2)\nu^2/C_1^2 C_2^2}.$$

We pin down the smaller root because in this case the price function will respond positively to decrease in the share of individuals investing in the market, that is, price increases when demand decreases:

$$a_3 = \frac{C_1 + 2C_2}{2C_1 + 2C_2} - \frac{\sqrt{\frac{\nu^4}{C_1^2 C_2^4} - 4\frac{\nu^2}{C_1^2 C_2^2}\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}}{2(C_1 + C_2)\nu^2 / C_1^2 C_2^2} =$$

$$= 1 - \frac{C_1}{2C_1 + 2C_2} - \frac{C_1\sqrt{1 - 4\frac{1}{\nu^2}C_2^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}}{2(C_1 + C_2)}.$$

Fraction $\frac{C_1}{2C_1+2C_2}$ is decreasing in δ (because $\zeta_0 \leq \zeta_1$) and $\sqrt{1 - 4\frac{1}{\nu^2}C_2^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}$ is increasing in δ; hence a_3 is an increasing function of δ for this root. Because price should respond negatively to falls in demand we end up with the other root,

$$a_3 = 1 - \frac{C_1}{2C_1 + 2C_2} + \frac{C_1\sqrt{1 - 4\frac{1}{\nu^2}C_2^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}}{2(C_1 + C_2)}. \qquad (A3.12)$$

This means that $A_{3,ii} = a_3$ where a_3 is given by $(A3.12)$.

Note that the right side of equation $(A3.9)$ is negative at $\widetilde{a}_3 = \frac{C_1}{C_1+C_2}$ and has a maximum at point $\overline{a}_3 = \frac{C_1+2C_2}{2C_1+2C_2} > \frac{C_1}{C_1+C_2}$; we conclude that the solution for $(A3.9)$ is greater than $\frac{C_1}{C_1+C_2}$, that is, $a_4 > 0$ from $(A3.8)$. Using $(A3.8)$ it can be shown that

$$A_{4,ii} = \frac{\frac{1}{\nu^2}C_1C_2\left(\frac{C_1+2C_2}{C_1+C_2} + \frac{C_1}{C_1+C_2}\sqrt{1 - 4\frac{1}{\nu^2}C_2^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}\right)}{C_1 + C_1\sqrt{1 - 4\frac{1}{\nu^2}C_2^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}} \qquad (A3.13)$$

Finally, by means of $(A3.5)$ we can derive

$$A_{1,ii} = \frac{1}{C_1 + C_2}[\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2} + \frac{\nu^2}{C_1C_2}\left(\frac{C_1}{2} + \frac{C_1}{2}\sqrt{1 - 4\frac{1}{\nu^2}C_2^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}\right)\right)^{-1} \times$$

$$\times \left(C_1\frac{\overline{Z}_i}{(\sigma_{ii}^Z)^2} - S_{0,ii}\right)], \qquad (A3.14)$$

$$A_{2,ii} = \frac{C_1}{(C_1 + C_2)(\sigma_{ii}^Y)^2}\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2} + \frac{\nu^2}{C_1C_2}\left(\frac{C_1}{2} + \frac{C_1}{2}\sqrt{1 - 4\frac{1}{\nu^2}C_2^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2}\right)}\right)\right)^{-1} \qquad (A3.15)$$

We have found the equilibrium given the share of individuals investing on their own. To make sure that the equilibrium indeed exists we have to check that there is a unique δ for any realizations of public signal, supply shock and final payoff. Private signals do not have an impact on equilibrium prices because private errors cancel out on average and δ depends on public information only.

To show that the equilibrium indeed exists I will show that coefficients of Y, Z, $\widetilde{S}$ and constant are decreasing in δ; then δ solving (33-35) will be unique. First, let's calculate coefficients of $\frac{1}{\delta}\Sigma_{pub}^{-1/2}(\widehat{Y}_{pub} - PR)$. If we show that this function decreases in δ then $\delta^2 t_{pub}$ will also decrease in δ. From $(A3.5)$ and $(A3.6)$ we get

$$A_1 = \frac{1}{C_1 + C_2}\Sigma_{pub}(C_1(\Sigma^Z)^{-1}\overline{Z} - S_0), \quad A_2 = \frac{C_1}{C_1 + C_2}\Sigma_{pub}(\Sigma^Y)^{-1},$$

$$A_3 = \frac{C_2}{C_1 + C_2}I + \frac{C_1}{C_1 + C_2}\Sigma_{pub}(\Sigma^p)^{-1}, \quad A_4 = \frac{1}{C_1 + C_2}(\Sigma_{pub} + C_1\Sigma_{pub}(\Sigma^p)^{-1}A_3^{-1}A_4).$$

We can also write:

$$\widehat{Y}_{pub} = \Sigma_{pub}((\Sigma^Z)^{-1}\overline{Z} + (\Sigma^Y)^{-1}Y + (\Sigma^p)^{-1}Z - (\Sigma^p)^{-1}A_3^{-1}A_4\widetilde{S}).$$

The equality means that:

$$\frac{1}{\delta}\Sigma_{pub}^{-1/2}(\widehat{Y}_{pub} - PR) = \frac{1}{\delta}\left[\frac{C_2}{C_1 + C_2}\Sigma_{pub}^{1/2}(\Sigma^Z)^{-1}\overline{Z} + \frac{1}{C_1 + C_2}\Sigma_{pub}^{1/2}S_0 + \frac{C_2}{C_1 + C_2}\Sigma_{pub}^{1/2}(\Sigma^Y)^{-1}Y\right] +$$

$$+\frac{1}{\delta}\left[\frac{C_2}{C_1 + C_2}\Sigma_{pub}^{1/2}(\Sigma^p)^{-1}Z - \frac{C_2}{C_1 + C_2}\Sigma_{pub}^{-1/2}Z - \frac{C_2}{C_1 + C_2}\Sigma_{pub}^{1/2}(\Sigma^p)^{-1}A_3^{-1}A_4\widetilde{S} + \frac{1}{C_1 + C_2}\Sigma_{pub}^{1/2}\widetilde{S}\right].$$

We should show that all the coefficients in this equality are decreasing in δ (assuming that realizations of Z, Y and S are positive). For that I rewrite the last equality as

$$\frac{1}{\delta}\Sigma_{pub}^{-1/2}(\widehat{Y}_{pub} - PR) = \frac{C_2}{\delta(C_1 + C_2)}\Sigma_{pub}^{1/2}(\Sigma^Y)^{-1}Y + \frac{C_2}{\delta(C_1 + C_2)}\Sigma_{pub}^{1/2}(\Sigma^p)^{-1}Z - \frac{C_2}{\delta(C_1 + C_2)}\Sigma_{pub}^{-1/2}Z$$

$$+\left(-\frac{C_2}{\delta(C_1 + C_2)}\Sigma_{pub}^{1/2}(\Sigma^p)^{-1}A_3^{-1}A_4 + \frac{1}{\delta(C_1 + C_2)}\Sigma_{pub}^{1/2}\right)(S_0 + \widetilde{S}) +$$

$$+\frac{C_2}{\delta(C_1 + C_2)}\Sigma_{pub}^{1/2}(\Sigma^Z)^{-1}\overline{Z} + \frac{C_2}{\delta(C_1 + C_2)}\Sigma_{pub}^{1/2}(\Sigma^p)^{-1}A_3^{-1}A_4S_0. \qquad (A3.16)$$

Note that we can rewrite:

$$\frac{C_2}{C_1+C_2}\Sigma_{pub}^{1/2}(\Sigma^p)^{-1} - \frac{C_2}{C_1+C_2}\Sigma_{pub}^{-1/2} = \frac{C_2}{C_1+C_2}\Sigma_{pub}^{1/2}((\Sigma^Z)^{-1} + (\Sigma^Y)^{-1})$$

and thus it is enough to show that $\frac{C_2}{\delta(C_1+C_2)}\Sigma_{pub}^{1/2}$ is a decreasing function of δ to prove the same for the coefficient of Z. Moreover, as we will see the only thing we have to prove is that $\frac{1}{\delta(C_1+C_2)}\Sigma_{pub}^{1/2}$ is a decreasing function of δ to prove that all the other coefficients are falling in δ.

To show that we need additional condition stated in the paper, namely (36).

Lemma 1. If (36) holds then $\frac{1}{\delta(C_1+C_2)}\Sigma_{pub}^{1/2}$ is a decreasing function of δ.

Proof of lemma 1. Consider the minus second power of this expression for element i on the diagonal:

$$\delta^2(C_1+C_2)^2(\Sigma_{pub,ii})^{-1} =$$

$$= \delta^2(C_1+C_2)^2\left(\frac{1}{(\sigma_{ii}^Z)^2} + \frac{1}{(\sigma_{ii}^Y)^2} + \nu^2\frac{1}{C_2^2}\left(1+\sqrt{1-\frac{4C_2^2}{\nu^2}\left[\frac{1}{(\sigma_{ii}^Z)^2}+\frac{1}{(\sigma_{ii}^Y)^2}\right]}\right)^2\right) =$$

$$= \delta^2(C_1+C_2)^2\left(\frac{\nu^2}{4C_2^2}\left[1+\sqrt{1-\frac{4C_2^2}{\nu^2}\left[\frac{1}{(\sigma_{ii}^Z)^2}+\frac{1}{(\sigma_{ii}^Y)^2}\right]}\right]\right). \qquad (A3.17)$$

To prove the lemma we have to prove that expression $(A3.17)$ is an increasing function of δ. Note that the factor $\left[1+\sqrt{1-\frac{4C_2^2}{\nu^2}\left[\frac{1}{(\sigma_{ii}^Z)^2}+\frac{1}{(\sigma_{ii}^Y)^2}\right]}\right]$ is an increasing function because C_2 is a decreasing function. Thus we have to prove that $\frac{\delta(C_1+C_2)}{C_2}$ is an increasing function.

Consider the first derivative of the logarithm of this fraction in δ:

$$\frac{1}{\delta} - \frac{1}{C_1+C_2}\left(\frac{1}{\rho_0(1-\gamma)} + \frac{\zeta_0}{\rho_0(1-\gamma)N}\right) + \frac{1}{C_2}\frac{\zeta_0}{\rho_0(1-\gamma)N} =$$

$$= \frac{1}{(C_1+C_2)C_2}\left[\frac{(C_1+C_2)C_2}{\delta} - \left(\frac{1}{\rho_0(1-\gamma)} + \frac{\zeta_0}{\rho_0(1-\gamma)N}\right)C_2 + \frac{\zeta_0}{\rho_0(1-\gamma)N}(C_1+C_2)\right] =$$

$$= \frac{1}{(C_1+C_2)C_2}\left[\frac{(C_1+C_2)C_2}{\delta} - \frac{a_0(\zeta_1-\zeta_0)}{\rho_0\rho_1 c(1-\gamma)N}\right].$$

The first summand in brackets decreases in δ and minimum of the expression in brackets is reached at $\delta = 1$:

$$\frac{a_0^2\zeta_1(1+\zeta_1/N)}{c^2\rho_1^2 N} - \frac{a_0(\zeta_1-\zeta_0)}{\rho_0\rho_1 c(1-\gamma)N} = \frac{a_0}{\rho_1 cN}\left[\frac{a_0\zeta_1(1+\zeta_1/N)}{c\rho_1} - \frac{(\zeta_1-\zeta_0)}{\rho_0(1-\gamma)}\right] > 0$$

by inequality (36). We see that the first derivative of $\frac{\delta(C_1+C_2)}{C_2}$ is positive on the interval $\delta \in [\gamma, 1]$ and thus the function is increasing. This proves Lemma 1. $\square$

Having proved Lemma 1 we can easily show that all the coefficients in $(A3.16)$ depending on δ are decreasing in it. First, because $\frac{1}{\delta(C_1+C_2)}$ is increasing in δ Lemma 1 shows that Σ_{pub} is decreasing in δ. Second, using (7) we conclude that $\Sigma^p = A_4^2 A_3^{-1}\nu^2$ is decreasing and combining with lemma 1 coefficient of S is decreasing as a sum of products of decreasing functions. C_2 is also decreasing and thus coefficient of S_0, Y, Z and $\overline{Z}$ are decreasing. This proves that overall right side in (33-35) is a decreasing function of δ.

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