Human Capital, Training and Portfolio Choice over the Life Cycle

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Abstract

The paper studies optimal labor, consumption, training and portfolio decisions in a life-cycle model with human capital and wealth accumulation. The agent can increase his future earnings by augmenting the human capital through training and learning-by-doing. The framework allows for retirement capital and i.i.d. process for employment and assumes two types of agents: constrained, who cannot invest into stock, and unconstrained. It is shown that the levels of wage income and the shape of wealth can be matched to the data, while the share of risky asset in the portfolio exhibit an inverse U-shaped form. Two special cases in which the agents either invest half of the wealth into risky stock or work for half of available time are studied. The agents who cannot flexibly adjust their labor or portfolio lose in consumption and wealth.

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1. Introduction

Human capital is the most important non-financial source of income. The wage streams associated with it and housing are the only sources of wealth for the households with no exposure to financial markets. Thus it seems interesting to investigate the behavior of agents who can influence their future incomes through human capital channel. This approach allows to endogenize earnings and consider simultaneous decisions on portfolio, consumption and labor choices.

There are three main ways to alter the human capital of an agent. First and the most important is to get a university degree or an MBA. I will not consider this type of education and will model it with the difference in initial human capital. The second way is "learning by doing" which means that a person can change his overall knowledge during working time. The last one is to have a short training courses, either general or specific to the job or firm in which the agent works. All three ways help to increase future streams of wage income and make earnings endogenous.

This paper examines the model in which the agent can choose his time of work and training, consumption and portfolio allocation. With a given probability, he can be unemployed in any period, and he maximizes the expected utility derived from consumption, leisure and final wealth. I assess the ability of the model to replicate general properties of US data on wealth and wage distribution, working time and share of risky asset in the portfolio.

Recent research has shown the ability of the life-cycle models to match US data. Yaron et al. (2003) studies the model in which agents with learning abilities fixed for the whole life can divide their available time between working for wage and increasing human capital. They set the wage per unit of human capital to be constant and thus endogenize the wage income because of changes in human capital. The paper shows that cross-sectional distribution of earnings can be matched closely for some initial distribution of abilities and human capital. Yet this paper does not consider consumption or portfolio choice and does not report the simulated labor supply. Moreover, the authors matched the behavior of the wage income from age 20 to age 58 and do not take into account sharp decline in earnings in the ages 60-65 which can not be explained by the model.

Shaw (1989) solves the model of leisure and consumption choice with quadratic function for human capital accumulation and no bequest. The wage is determined from equilibrium conditions. She estimates the parameters of the model using US data and show that rental rates of human capital should be changing over time to match the labor and investment in human capital choices.

Gourinchas and Parker (2002) consider the framework with consumption choice in which they are able to characterize consumption over the life cycle for different groups of consumers taking into account their education. The income process is exogenous and stochastic. The authors show that households save actively in later periods while enjoy consumption on earlier stages of their life. They argue that this behavior is incompatible with benchmark models of representative consumer.

French (2005) builds the model to study labor supply and retirement behavior. He allows the agents to differ in their health and to choose consumption and leisure. The process for wages is stochastic, depends on individual's characteristics and includes autoregressive part. The author shows that wealth accumulation is close to what observed in the US data but the shape of the labor supply in earlier ages can not be reproduced.

Cocco, Gomes and Maenhout (2005) study the models in which wage income is an exogenous yet realistically calibrated process. In these models the agent chooses consumption and portfolio, and wage income plays the role of riskless asset because its return is uncorrelated with the equity return. The findings support common rule of decreasing share of risky asset in the portfolio and generate consumption profiles reasonably close to US data.

Ameriks and Zeldes (2004) and Wachter and Yogo (2009) provide empirical evidence on portfolio share dependence on wealth and age. It is shown that in Survey of Consumer Finance the share of risky assets in the portfolio rises with wealth and is non-decreasing in age.

Finally, Gomes, Kotlikoff and Viceira (2008) consider flexible labor supply and portfolio choice model in which leisure and consumption are substitutes and utility is non-separable in these two variables. Authors include retirement into the model and wage follows exogenous stochastic process. The shapes of labor supply and consumption correspond to those found in data. Portfolio share behaves as a decreasing function of age until retirement and increases afterwards.

My model is similar to Yaron et al. (2003) in human capital process and differs in three dimensions. First, the agents can use training to raise human capital. This allows the agents to accumulate knowledge faster in younger and middle ages and contributes into the increase in variation in wage income with age. Second, I assume that the agent can be unemployed with i.i.d. rate in any period. This feature of the model influences the resulting share of the risky asset in portfolio and makes it inversely U-shaped rather than decreasing with age. Third, I allow for portfolio and consumption choice.

The main results of the paper are the following. I show that the levels and shapes of wealth and wage income can be matched. The labor supply is a decreasing function of age and the level of training shows inverse U-shaped behavior. Portfolio holdings exhibit an inverse U-shape, while consumption is an increasing function of age but is almost flat near retirement. Finally, it is shown that the results are qualitatively robust to changes in parameters.

In the second part of the paper I set up the model and shortly discuss the solution tools. In part 3 I consider the data and the parameters for calibration. Part 4 summarizes the results, and part 5 states some robustness check. Conclusions finalize the paper in part 6.

2. The Model

2.1. Assets, Wealth and Investment Opportunities

Agents have two types of assets: financial wealth X_t and non-tradable human capital H_t which generates wage income.

There are two financial securities in the economy: a risk-free bond with return R_f and a risky stock with log-normally distributed return R_t. I assume that returns on risky asset are i.i.d. over time periods. The agent decides on the portfolio share of the riskless asset $(1-\alpha_t)$ and α_t is invested in stock.

The agent also receives wage income. I assume that labor market is competitive. It means that the agent is paid the wage W_t for the unit of human capital supplied to the labor market; hence, his income is equal to $Y_t = W_t H_t L_t$. Here L_t is the labor supply. Wage is assumed to follow Markov chain with just two states. One of the states is set to be 0 to capture the effect of possible unemployment of the agent and the other depends on time (see section 3 for details). I call these two states "unemployment" and "employment" respectively.

Finally, human capital can be increased in two ways. First, the agent experiences "learning-by-doing" and his human capital improves with more time spend on the labor market. Second, he can train his abilities and augment the future expected income stream, yet this is costly activity both because of fall in leisure and direct financial costs (the costs are included into the budget constraint (2)). I denote by E_t the training time. The evolution of human capital is assumed to be

$$H_{t+1} = e^{a\beta_H E_t^{\lambda_H}}((1-\delta_t)H_t + aL_t). \tag{1}$$

Here δ_t is the rate of depreciation of human capital and a is the learning ability of the agent; the difference in the latter across agents is shown to be important to explain cross-sectional distribution of wage income in Yaron (2003). As in that paper, I consider a to be fixed for the agent for the whole life cycle.

Compare (1) to what Yaron et al. (2003) uses: in their paper the evolution of human capital is

$$H_{t+1} = (1-\delta)H_t + a(H_t(1-L_t))^\eta.$$

The difference is that, first, the "training" or production of human capital $a(H_t(1-L_t))^\eta$ is not a proportional factor which it appears to be in econometric analysis (see discussion of parameters in section 3) and the returns to human capital are higher for a given level of labor supply in Yaron's model. Second is that time spent on the work is not considered to increase human capital. The final difference which has to be emphasized is that in Yaron's paper the human capital depreciation rate δ is constant over time while in my model it is different in the first and the second half of the life cycle.

Training costs depend on the state of wage: it equals c_e for the unit in employment state and c_u for the unit in unemployment state. Denote the costs of training as $c(W_t)$.

The timing of the model is the following. In the beginning of period t the agent knows his wealth X_t, human capital H_t, learning abilities a and wage W_t and chooses portfolio allocation, labor, training and consumption. The agent observes the borrowing constraint and cannot borrow using future income as a collateral, thus $C_t \leq X_t + Y_t$. In the end of the period return on equity is realized.

The budget constraint for financial wealth is then:

$$X_{t+1} = (W_t H_t L_t + X_t - C_t - c(W_t)E_t)(\alpha_t R_t + (1-\alpha_t)R_f). \tag{2}$$

We do not include tax on labor income because this only changes the scale of W_t.

2.2. Agents and Preferences

We assume that there is a number J of agents who live for a fixed number of periods T. The agents are different in their learning abilities, initial levels of human capital, financial wealth and investment opportunities. Agents $j = 1, ..., J_1$ can invest into a risky stock, while agents $j = J_1 + 1, ..., J$ can not.

Consider first an agent $j = 1, ..., J_1$ who has a given learning ability (a), initial financial wealth ($X_0 > 0$) and human capital ($H_0 \geq 0$), and is able to invest into a risky asset. Each period the agent is endowed with 1 unit of time and maximizes expected utility which is separable in consumption and leisure:

$$\max \mathbf{E}_1 \left[\sum_{s=1}^{T} \beta^s \left(\frac{C_s^{1-\gamma}}{1-\gamma} + \nu \frac{(1 - L_s - E_s)^{1-\lambda_L}}{1-\lambda_L} \right) + \kappa \beta^{T+1} \frac{X_{T+1}^{1-\gamma}}{1-\gamma} \right]. \tag{3}$$

Here C_s is the consumption at time s whereas L_s and E_s are labor and training chosen at time s, and we set lower bound for leisure such that $L_s + E_s \leq \overline{L}$. $V(X_{T+1}) = \kappa \frac{X_{T+1}^{1-\gamma}}{1-\gamma}$ captures bequest motives and retirement wealth.

The agent maximizes (3) taking into account (1), (2) and "no default" constraint:

$$X_t \geq 0. \tag{4}$$

Agent $j = J_1 + 1, ..., J$ is unable to invest in a risky asset and he solves the same problem except his share of stock is always 0, $\alpha_t \equiv 0$. For this type of agents randomness is generated by changes in wages. From now on we do not differentiate two types of agents unless it is stated explicitly.

2.2. Solution of the model

The problem can not be solved analytically and is solved numerically using backward induction and Bellman equation.

In the end of the last period the agent has utility $V(X_{T+1})$ and solves the problem in the beginning of period T with constraints (2-4):

$$V_T(X_T, H_T) = \max_{C_T, E_T, L_T, \alpha_T} \left(\frac{C_T^{1-\gamma}}{1-\gamma} + \nu \frac{(1 - L_T - E_T)^{1-\lambda_L}}{1-\lambda_L} + \beta \mathbf{E}_T V(X_{T+1}) \right). \tag{5}$$

The solution is trivial in E_T, since V does not depend on H_{T+1} and thus $E_T = 0$.

In period $s = 1, ..., T-1$ the agent solves the problem with constraints (2-4)

$$V_s(X_s, H_s) = \max_{C_s, E_s, L_s, \alpha_s} \left(\frac{C_s^{1-\gamma}}{1-\gamma} + \nu \frac{(1 - L_s - E_s)^{1-\lambda_L}}{1-\lambda_L} + \beta \mathbf{E}_s V_{s+1}(X_{s+1}, H_{s+1}) \right). \tag{6}$$

The model is solved using grid search, Gaussian quadrature and piecewise shape-preserving cubic interpolation in X_t and linear interpolation in H_t.

3. Data and calibration

3.1. Data

I use two databases to construct the proxies for wealth, wage income, labor and training. The Panel Study of Income Dynamics (PSID) supplies statistics on wealth, labor time and wage income for several years from 1983 to 2004 (I use data for years 1983, 1988, 1993, 1998, 2000, 2002 and 2004 due to availability of the data on wealth in these years). The same data was also studied by Yaron (2003). The total number of observations used in my paper is 8035 for stockholders and 29151 for non-stockholders which is roughly 865 per year. The other database is National Longitudinal Survey of Youth 1979 (NLSY) which has multiple training questions, namely the length of the training. This data is available for ages 23-49 only. In NLSY the number of those who receive training equals 11148 out of 115543. I will set the upper bound for the training to be 400 hours per year and thus the number of observations used to generate first two moments for training is equal to 10024.

I consider the following variables as a proxies for the model variables.There are two wealth proxies I take into account. The first one is the sum of stock and bond holdings. The second one is wide wealth measure including housing and business. For now I abstract from the cohort effect and consider raw moments of the data. Training proxy is the total time spent on four technical/vocational training courses in NLSY79. This variable is shown to have the most effect on wage increase in Frazis and Loewenstein (2004). I also consider the shape of training reported in the Organization for Economic Co-operation and Development "Economic outlook" (1998) pp.139-140: the share of workers who receive professional and career-upgrading training in US has a peak at 45-55, slightly decrease afterwards and is increasing from 25 to 45.

Figures 1-3 show statistics for labor, training, financial wealth (I do not show total wealth), wage income and portfolio share invested in risky asset over the life cycle. Financial and total wealth and wage income are calculated for the respondents of PSID who possess more than 100 and less than 500000 dollars of financial wealth in year 1984 real terms. On figure 2 these variables are reported in 2006 US dollars. Interestingly, the mean of training is almost flat for all the ages available in NLSY79. I put right axis on figure 3 to express number of hours of training per year. The average number of hours spent on training is close to 74 per year over the ages 23-49 and the standard deviation of this mean is approximately 7 (if we exclude two outliers at ages 23 and 48). Labor is restricted to be between 400 and 4000 hours per year. We observe an almost flat labor supply and variable wage and wealth.

Figure 4 reports the proportion of agents who invest in risky asset. This share is increasing over the life cycle (see figure 4) and average to 0.3 in PSID. The only variable which can not be explicitly defined for each group of agents is training (because it is not present in PSID) and I assume that the fraction of stockholders in the whole population is 0.3.

[Insert here figures 1-4].

3.2. Parameters: returns, utility function and discount rate

The time period in the model is one year. There are $T = 41$ periods, the agent starts his life cycle at age 23 and retires at the beginning of age 66.

The logarithm of a risk-free rate is 1% and risk premium is 4%. Discount rate is $\beta = 0.96$.

The parameters of the utility function for the unconstrained agents in the benchmark model are chosen in such a way that the share of a risky asset is reasonably high and the labor supply is closer to the flat one. I set $\gamma = 8$ and $\lambda_L = 7$ to achieve both goals. Then $\kappa = 20000$ is such that the distribution of wealth is similar to that in the data and $\nu = 3$ is chosen to match average labor supply. We can rewrite the retirement function as $V(X_T) = 43.6\frac{(X/2.4)^{1-\gamma}}{1-\gamma}$ for this parametrization; the average final wealth will be around 2.4 for the unconstrained agents and hence κ is not very high given this scale.

The constrained agents have slightly different utility function, namely I change κ to reflect the fact that consumption streams should be reasonably high and average labor supply is lower. If κ is the same as for the unconstrained agents, the constrained agent will save more to meet the requirement to have higher retirement wealth and will work more. However, in the data non-shareholders work less on average than stockholders. To achieve lower wealth accumulation and lower labor supply, I set $\kappa = 3000$ for the constrained agent.

3.3. Parameters: labor and training

The (real) wage for employed is set to be equal to $W_T = 0.2$ in the last period (for scaling purpose) and is equal to $W_t = W_t/(1+g)^{T-t}$ where $g = 0.0014$ is chosen to fit PSID growth rate for average real earnings per working person. Since there are two ways to increase human capital and we observe decrease in real income in the end of life cycle, the depreciation rate of human capital is set to be higher than in Yaron (2003) and is equal to 4% on average. In the benchmark model I set $\delta_t = 0.02$ for $t = 1, ..., 22$ and $\delta_t = 0.06$ for $t = 23, ..., 41$.

This choice of the parameter δ in the benchmark model makes the decrease in human capital faster in older ages and allows to generate correct shape of wage income. Yet it is disputable and for the robustness check I solve the models with flat depreciation rate and show that the shape of earnings can not be replicated. In the model with flat rate the peak of wage income is in the fifties rather than in the forties, and this is the result of the proceeding human capital raise in the second part of the life cycle.

There are 90 hours per working week the person may spend on work, training and leisure (we exclude 30 hours a person may spend on sleep). The lower bound of leisure is set at $\overline{L} = 0.1$ (that is, 27 hours) and thus the agent cannot work and train more than $4500 \times 0.9 = 4050$ hours per year. Average labor supply over life cycle is around 2125 or 0.48. The upper bound for labor is 0.8.

I consider training time to take no more than 0.098 of total time which add up to 400 hours per year. This is a reasonable upper bound since in the NLSY79 sample maximum training time is less than 400 for almost 90% of the sample.

I set $\lambda_H = \frac{1}{3}$ as in Frazis and Loewenstein (2004). This paper and several others (Bartel (1995), Booth, Bryan (2005)) have shown that average increase in wage for 60-100 hours training is $4-6\%$; I set $\beta_H = 0.4719$ because higher constant leads to very high levels of training. With this parametrization, 74 hours of training (the average number of training hours in NLSY79) means 6% increase in wages for the agent with the learning ability $a = 0.5$.

The cost of training is more tricky because most of the training is paid by the employer (or government if a person in unemployed), but there is some evidence (Loewenstein, Spletzer (1998), Booth, Bryan (2005)) that during the training time a person either has lower wage because spends less time on work, no wage if he trains before the job or just shares costs with the employer. Moreover, staying with the same employer reduces the returns on training in comparison to changing the employer after the training. This means that our assumed increase in wages is higher than in reality because there is no change of work in the model. Thus, to generate realistic levels of training I consider explicit cost of training and set it to be $c_e = 1.5$ (or 2500 dollars in year 2006 for the training of 74 hours) and $c_u = 0.1$. The parameters are summarized in the table 1.

Table 1. Parameters of the Model

Parameter	Value
Discount rate (β)	0.96
Utility function parameter (γ)	8
Utility function parameter (λ_L)	7
Retirement wealth parameter for unconstrained (κ)	43.6
Retirement wealth parameter for constrained (κ)	43.6
Wage increase parameter (β_H)	0.4719
Training power parameter (λ_H)	1/3
Cost of training for employed (c_e)	1.5
Cost of training for unemployed (c_u)	0.1
Leisure utility parameter (ν)	3
Human capital depreciation (δ)	0.02; 0.06
Share of agents able to invest in risky asset	0.30

3.4. Calibration

The main goal of the paper is to match first moments of five variables: wealth X_t, wage income $W_t H_t L_t$, training time E_t, labor time L_t and portfolio share α_t. I concentrate on financial wealth and do not take into account total wealth of the agent which includes housing and durables. The total wealth is approximately two times higher than financial wealth in PSID.

The levels of these variables are shown in figures 1-4. I summarize the levels for different age groups in tables 3-4 where I compare simulated moments to the data. Mean of the total wealth is reported in tables 3-4 in the column W_1. It would be better to report financial variables as a ratio to earnings at the same age yet I report them in 2006 US dollars for simplicity and to make comparison more clear.

I solve the model for a range of possible values of learning ability: $a = 0.3, 0.35, ..., 0.5$ for unconstrained agents and $a = 0.2, 0.25, ..., 0.4$ for constrained agents. This range of parameters is chosen because of the following arguments. First, the agents with lower abilities would have low return on training since the increase in wages for the agent with ability $a = 0.3$ spending 74 hours on training is 3.4% which is low enough. Thus the training will be too expensive for them and they will not train at all. Second, the agent with ability lower than $a = 0.2$ work too much on average, his average working time over the life cycle is close to 0.6. These agents are unable to increase their human capital by means of training, they have low return on learning-by-doing and hence they have to work more to produce reasonable streams for consumption and wealth. Yet this high labor supply contradicts the data.

Then I simulate the behavior of all the variables. For a given a, initial level of wealth is $X_0 = 0.2$ for agents who invest in a risky asset (that is,approximately 30000 dollars which corresponds to PSID data) and $X = 0.02$ for agents who do not invest in stock. Also, I start from different levels of human capital which depend on learning ability. For a given a, all the pairs (a, H) have the same probability. I assume that for higher level of learning ability a the initial level of human capital is higher on average. This assumption is plausible for two reasons. The agent with the higher ability to learn may have better pre-work education. Moreover, as will be seen later, the agents with high abilities acquire high levels of human capital during the life cycle. Even if they start with lower initial human capital, they end up with higher human capital and wages. Yet in the data the order is preserved: these who earn more in the middle and older ages tend to earn more in younger ages. This means that the agents with better learning abilities tend to have higher initial human capital.

Consistent with figure 2 I assume that constrained agent is of lower ability and initial human capital on average.

The exact pairs (a, H) are shown in the table 2. The choice for the stockholders is motivated by the fact that initial wage at age 23 is around 30000 dollars, so that around 0.2 in terms of the model and that the maximum wage at ages 38-56 is approximately 2.5 times higher (in US 2006 dollars) than initial wage at age 23. Combining these facts, the goal is to set initial levels of human capital such that the behavior of the earnings is inversely U-shaped with a peak at ages 40-49.

The choice for non-stockholders is similar, but I set even lower initial human capital because the wage is lower for this category (25000 dollars at age 23).

Table 2. Distribution of Learning Abilities and Human Capital

Value of a	Values of H, Unconstrained	Values of H, Constrained
0.2	-	0.9, 1.1
0.25	-	0.9, 1.1
0.3	1, 1.2	1, 1.2
0.35	1.2, 1.4	1.2, 1.4
0.4	1.4, 1.6	1.4, 1.6
0.45	1.6, 1.8	-
0.5	1.8, 2	-

4. Results for the benchmark model

The results for unconstrained agents are shown in the table 3 and the results for constrained agents in the table 4. I report average levels of the variables over the life cycle and their PSID and NLSY proxies. I express financial variables in terms of 2006 dollars to compare with the data. Labor and training are written in fractions of total available time. There is no plausible proxy for the consumption in PSID so that I do not report it and do not report standard deviations in the simulations. The standard deviations are expressed in the same units as the variables.

The figures 5-8 show the average behavior of the respective variables for both types of agents.

Figures 5 and 6 show the behavior of the wage income and wealth. Financial wealth is accumulated more aggressively in the model than in the data, but this behavior is partially explained by the fact that I do not take into account other types of wealth, for example housing. Total wealth including non-financial wealth is approximately two times higher than financial wealth in PSID and simulated wealth is close to the total wealth. The overall accumulation of wealth repeats the shape of that in the data until the last period of life (ages 56-65) when the agents in the model start de-saving. This result is in line with previous research, for example Gomes, Kotlikoff and Viceira (2008).

The wage income exhibits inverted U-shape as in the data and matches the data closely. The agents increase their human capital and earnings sharply until earlier fiftieth and start to acquire less training and learning after that age. Earnings increase slower in the ages 23-30 than in the data and behave well in the last periods.

The main caveat is consumption. In the utility function, consumption and leisure are compliments to some extent; this means that higher leisure leads to higher consumption unlike in Gomes, Kotlikoff and Viceira (2008). This result contradicts the summary in Gourinchas and Parker (2002), yet consumption still shows reasonably flat behavior in the end of the life cycle and does not increase substantially. Also, consumption increases over the life cycle which is consistent with the evidence reported in French (2005) and Gourinchas and Parker (2002).

Figure 7 reports the average share of risky security in the portfolio. The model generates too high share of risky asset in the middle ages: while in PSID data the share is close to 0.5 and never reaches 0.6 in the middle ages, simulated proportion of financial wealth in equity is increasing until ages 51-60. Moreover, the share is above 0.7 for the agents with higher learning ability and higher initial level of human capital.

Nevertheless, this result should not evoke much of concern. The inverted U-shape of the risky share in the portfolio is related to the fear of simultaneous unemployment and bad portfolio return which preserves low alpha for the beginning and the end of the life cycle. At the same time, agents with higher abilities and higher wealth begin to invest more into risky asset because they can cover their consumption in bad times until they become employed and they are eager to collect higher returns on investment. The result is actually close to what is shown in Ameriks, Zeldes (2004) and Wachter, Yogo (2009): the share of risky asset is increasing in wealth and non-decreasing in age. In my model wealth and age are closely related by means of human capital accumulation and thus inverted U-shape pattern is reasonable.

Training and labor are reported in figure 8. Training is not matched closely and follows inversely U-shaped pattern but this is mostly because of the fact that the training is not very volatile over the ages in NLSY and has approximately the same mean for all ages. Yet the share of agents receiving the training increases with age in the model which reflects the ability of the agents to give up a part of consumption and leisure for future earnings. The younger the worker the less wealth he possesses and the less he is willing to spend time on additional activities except working. This pattern is somehow disputable because Mincer (1989) and Loewenstein, Spletzer (1998) mention that the agents have more training in younger ages than in older ones, but the share of workers who receive training is almost the same as in NLSY in all the ages with the mean 8.7% and standard deviation of only 0.6%. Dearden, Reed and Van Reenen (2005) mention that the share of workers receiving training is around 14% in the first half of the 1990th, and Bartel (1995) has a sample with 50% attendance of training. This means that the actual share of trained workers and the hours they train is higher than in NLSY. The study published by Organization for Economic Co-operation and Development (1998) shows the shape the model predicts: the training participation peaks for workers at ages 45-54 and slightly decreases for workers at ages 55-65.

Moreover, learning-by-doing involved in the model allows employees to improve their skills even without formal training and thus the number of formally trained agents decreases compared to the model with no learning-by-doing. The overall behavior of the agents is rational: they raise human capital by learning on work until they find it impossible and then start to acquire paid training.

The labor in the model does not behave so close to the data and is too high in the first periods, but qualitatively the results are close to what is shown by French (2005).

The shape of this variable is not replicated, but the overall decrease in labor supply is the same as in PSID.

Second moments of the data are matched to a lesser extent. Wage income and wealth have rising standard deviations until the age 60 but are lower than in the data, and most of the other second moments are also low. Interestingly, the volatilities of labor and training are matched better than the ones of the financial variables. The reason for lower standard deviations in the model than in the data is that the abilities of the agents are very close to each other and thus lead to a small variance in financial variables. Despite that, the overall behavior of variances is like in the data: the difference between the agents increases over time until the retirement age.

5. Robustness check

To understand what drives the results of the model I implement two robustness checks for unconstrained agents in the benchmark model. The first one is to set $\alpha = 0.5$ and consider flexible labor supply. The second one is to set $L = 0.5$ (that is, the average labor supply for the whole life cycle) and to derive optimal policy rules under this assumption.

Then I change the utility function to see the dependance of the variables on parameters γ and λ_L. The model is solved for the following parameters of the utility function: $\gamma = 5, \lambda_L = 3$, $\gamma = 8, \lambda_L = 3$, $\gamma = 5, \lambda_L = 7$. The coefficients κ and ν are changed to preserve the same average labor supply for the life cycle and similar consumption streams. I do not report them here.

Finally, the model with flat $\delta = 0.04$ is considered. The other parameters are the same as in the benchmark model.

Tables 3C and 3D contains the optimal solutions for the benchmark model with $\alpha = 0.5$ and $L = 0.5$. We see that the agent who can not change his labor supply is worse off getting lower wage and consumption stream than the unconstrained agent. He accumulates more wealth at the end of the life but has lower utility of leisure near the retirement age. The result is mostly the fact of lower training and share of risky asset in the portfolio: the agent acquires less wealth in younger ages, he is less willing to train in the middle ages and thus end up with the peak of wage at fifties, lower consumption stream and higher wealth at the end of life cycle.

The agent with constant share of risky asset, however, follow very similar dynamic as the one with no constraints, and the reason is that the average share of risky asset in the portfolio is very close to 0.5 in the benchmark model. The labor supply behaves closer to the data though still misses the correct shape. Wealth is accumulated until the retirement because of fixed share of risky asset.

The results for different parameters of the utility function are shown in table 4. For both models with $\gamma = 5$ the average levels of wealth and wage income seem to be similar to the benchmark model, but the shape of the earnings is skewed to the end of the life cycle, that is, the peak values are produced in the late fifties or early sixties. The share of risky asset, as expected, is higher and close to 1 in the middle ages due to lower risk aversion, and labor supply is more flat than in the benchmark model when $\lambda_L = 7$ but falls more sharply when $\lambda_L = 3$. Training behaves similar to the baseline results and wealth accumulation is more aggressive due to higher share of risky asset in the portfolio. Consumption exhibit better pattern than in the benchmark model and slightly decreases at the end of the life cycle.

For $\gamma = 8$ and $\lambda_L = 3$ the results are very close to the benchmark model, but the difference is nonetheless significant. First, the shape of wage income does not follow the data and has a peak in fifties rather than in forties. Second, labor supply declines sharply and matches the data poorly.

Finally, I report one model with flat $\delta = 0.4$ for unconstrained agents (the results for different utility function parameters and constrained agents are qualitatively similar). In this case the peak of earnings is in fifties or sixties and wealth is increasing until the retirement. Thus flat depreciation rate can not produce correct shape of wage income in the model.

6. Conclusions

In this paper the model of human capital accumulation and portfolio choice is studied for constrained and unconstrained agents. It is shown that the model can produce qualitatively and quantitatively correct shapes and levels of wage income and wealth. I find that the model generates reasonable patterns for consumption and labor supply. Both share of risky asset in the portfolio and training exhibit inversely U-shaped form. The differences in wages are partially captured by the difference in abilities of the agents. The results of the model may be important to understand individual behavior of financial and labor decisions.

There are several possible ways to improve the model. First, the model assumes that the wage rate is constant and thus the labor market is competitive. It is interesting to include other types of wage process to reflect business cycle features. The retirement income paid at retirement age can also be included and this may produce more flat behavior of labor supply.

Second, most of the training is explicitly paid by the employer. We may consider the model in which the costs of training are low or even zero. The way to make the agents getting reasonable levels of training hours may be to assign the possibility to train randomly or to assume that the returns to training are random. In this case the agent will choose training time taking into account the loss in leisure and possibility of having no increase in human capital. We can also consider the model with risky human capital which may change the overall behavior of labor and training.

Third, to make consumption and leisure substitutes one can incorporate the utility function used in Gomes, Kotlikoff and Viceira (2008). This approach would help to reduce consumption in the end of the life cycle and at the same time change the shape of labor supply in the beginning of life. However, this may not solve the problems with labor supply if learning-by-doing is possible: starting the work the agent will try to increase his future earnings and thus work more.

Finally, the model may allow for more realistic unemployment rates and include possibility of death or labor inability. This may also change the labor supply and training choice because the agent will take into account the randomness of future earnings.

Table 3. Results for the unconstrained agents

A. Data

	Stockholders, mean						Stockholders, std				
Age	W_1	X	α	L	E	Y	X	α	L	E	Y
23-30	78173	35980	0.531	0.482	0.016	43940	58779	0.297	0.134	0.016	28666
31-40	199833	80582	0.556	0.506	0.016	68258	133651	0.299	0.116	0.016	51976
41-50	293753	146065	0.532	0.498	0.016	75940	198402	0.304	0.122	0.016	58650
51-60	428738	199037	0.512	0.473	0.016	71189	232081	0.307	0.130	0.016	60486
61-65	567382	212514	0.513	0.418	0.016	57659	254816	0.319	0.145	0.016	49602

B. Benchmark model

	Model, mean					Model, std				
Age	X	α	L	E	Y	X	α	L	E	Y
23-30	52669	0.260	0.735	0.00	39794	12656	0.032	0.027	0.00	6091
31-40	125370	0.390	0.574	0.009	58027	37507	0.093	0.052	0.006	9251
41-50	232224	0.690	0.447	0.019	75001	103192	0.17	0.088	0.008	14841
51-60	366383	0.577	0.383	0.017	70889	170805	0.202	0.118	0.005	17411
61-65	355898	0.344	0.342	0.005	57975	138964	0.046	0.148	0.001	17411

C. Fixed labor supply ($L = 0.5$)

	Model, mean					Model, std				
Age	X	α	L	E	Y	X	α	L	E	Y
23-30	32319	0.282	0.5	0	24066	3536	0.064	0	0	4625
31-40	69984	0.300	0.5	0.004	40208	6803	0.037	0	0.003	8825
41-50	161051	0.423	0.5	0.009	65474	15621	0.160	0	0.004	22784
51-60	357865	0.436	0.5	0.007	71022	23955	0.254	0	0.002	27389
61-65	482307	0.286	0.5	0.003	65102	33465	0.142	0	0.002	23190

D. Flexible labor supply, fixed share of risky asset ($\alpha = 0.5$)

Model, mean						Model, std				
Age	X	α	L	E	Y	X	α	L	E	Y
23-30	53414	0.5	0.728	0.00	39344	14934	0	0.046	0.00	6656
31-40	129829	0.5	0.572	0.010	57500	40167	0	0.056	0.007	9264
41-50	216497	0.5	0.458	0.021	74166	74924	0	0.082	0.010	15059
51-60	329557	0.5	0.411	0.016	72971	125447	0	0.114	0.007	16922
61-65	344385	0.5	0.355	0.005	59338	120269	0	0.132	0.002	17025

Table 4. Results for the constrained agents

A. Data

Non-stockholders, mean						Non-stockholders, std			
Age	W_1	X	L	E	Y	X	L	E	Y
23-30	41076	6923	0.469	0.016	33034	17368	0.129	0.016	18020
31-40	87437	13910	0.488	0.016	44432	35120	0.121	0.016	29831
41-50	138106	25449	0.489	0.016	49151	58963	0.122	0.016	36821
51-60	198448	45235	0.467	0.016	46799	89984	0.127	0.016	34616
61-65	233138	49971	0.411	0.016	36354	103748	0.144	0.016	28317

B. Benchmark model

Model, mean						Model, std				
Age	X	α	L	E	Y	X	α	L	E	Y
23-30	33999	0	0.708	0.001	29245	8532	0	0.032	0.001	4369
31-40	82967	0	0.544	0.008	40873	23244	0	0.058	0.008	6476
41-50	130265	0	0.456	0.015	49714	32457	0	0.078	0.015	8545
51-60	171434	0	0.427	0.010	46681	45255	0	0.088	0.010	9457
61-65	160564	0	0.389	0.003	38818	46643	0	0.106	0.003	6823

Table 5. Robustness check

A. $\gamma = 5,\ \lambda_L = 7$

Unconstrained agent						**Constrained agent**			
Age	X	α	L	E	Y	X	L	E	Y
23-30	41655	0.412	0.609	0	39672	25802	0.582	0.001	29011
31-40	86763	0.673	0.523	0.008	59450	57917	0.490	0.008	41996
41-50	180532	0.908	0.437	0.020	81575	95248	0.427	0.015	53276
51-60	328249	0.853	0.402	0.019	85028	137848	0.426	0.012	53713
61-65	433831	0.470	0.391	0.005	79410	182593	0.424	0.004	50483

B. $\gamma = 8,\ \lambda_L = 3$

Unconstrained agent						**Constrained agent**			
Age	X	α	L	E	Y	X	L	E	Y
23-30	54381	0.268	0.777	0.00	42041	34384	0.725	0.002	29162
31-40	140134	0.513	0.589	0.013	60796	78647	0.514	0.014	37836
41-50	267362	0.881	0.416	0.021	71793	135815	0.421	0.021	46385
51-60	367777	0.732	0.320	0.014	58788	165944	0.344	0.015	35928
61-65	286783	0.406	0.289	0.004	44125	117625	0.297	0.003	25697

C. $\gamma = 5,\ \lambda_L = 3$

Unconstrained agent						**Constrained agent**			
Age	X	α	L	E	Y	X	L	E	Y
23-30	40890	0.451	0.724	0.00	39864	36156	0.689	0.002	28922
31-40	95220	0.771	0.575	0.012	59397	82701	0.536	0.014	41423
41-50	202749	0.978	0.445	0.022	78134	142816	0.466	0.020	57013
51-60	347668	0.927	0.358	0.018	71234	174497	0.381	0.017	48921
61-65	318674	0.559	0.340	0.005	59743	123688	0.334	0.003	38466

D. $\gamma = 8,\ \lambda_L = 7,\ \delta = 0.04$

Unconstrained agent					
Age	X	α	L	E	Y
23-30	50924	0.257	0.740	0.00	38330
31-40	112272	0.366	0.608	0.007	51984
41-50	185833	0.552	0.507	0.016	63909
51-60	289247	0.516	0.429	0.013	71589
61-65	326900	0.359	0.356	0.005	67087

Figure 1. Share of The Risky Asset in the Portfolio

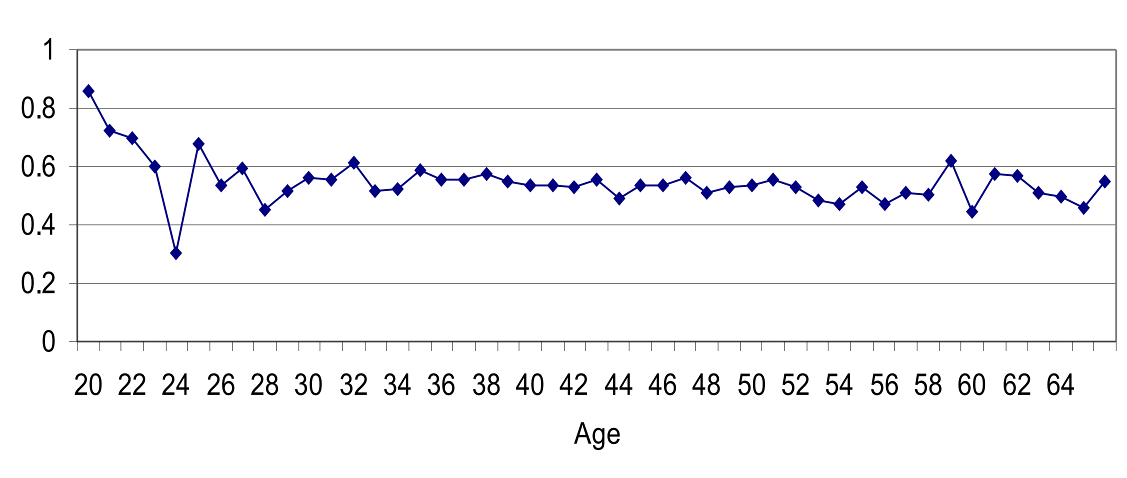
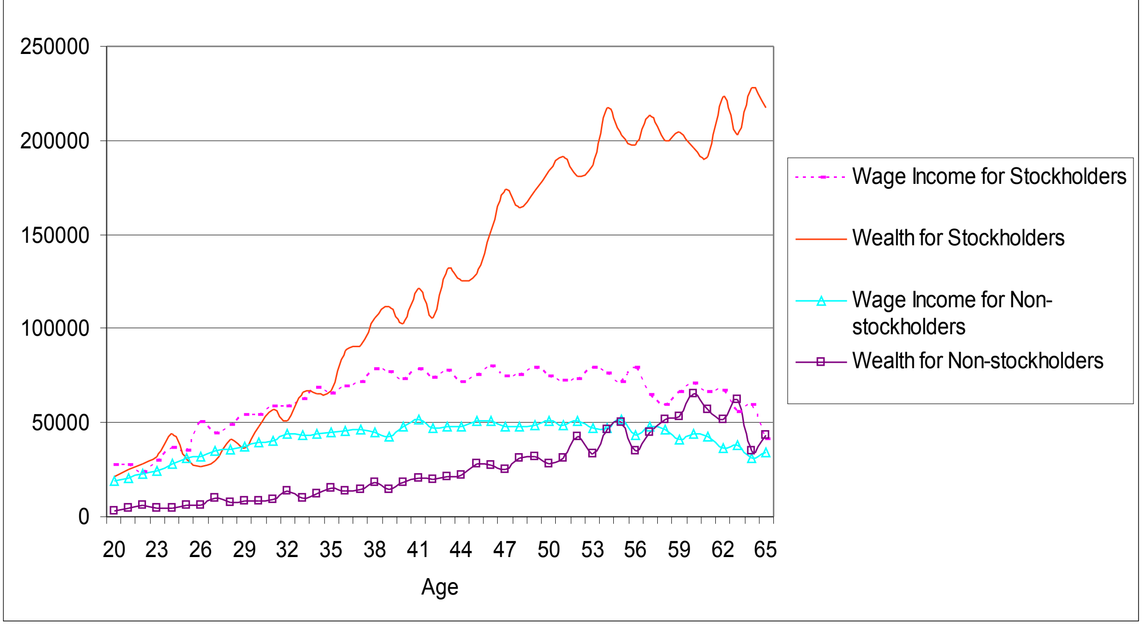


Figure 2. Financial Wealth and Wage income



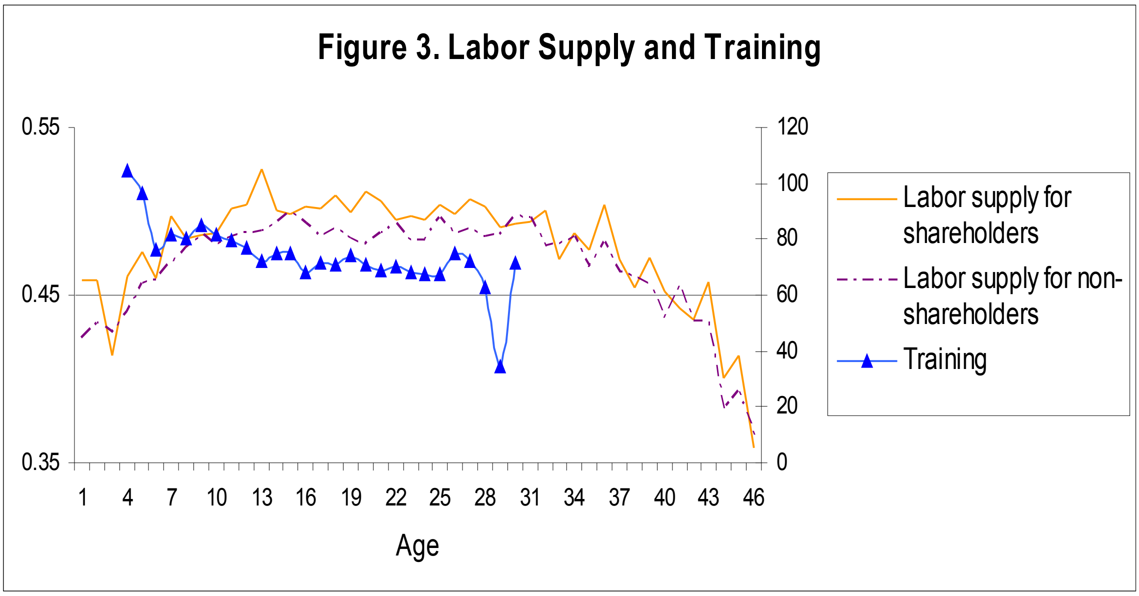


Figure 3. Labor Supply and Training
0.55
0.45
0.35
120
100
80
60
40
20
0
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46
Age
Labor supply for shareholders
Labor supply for non-shareholders
Training

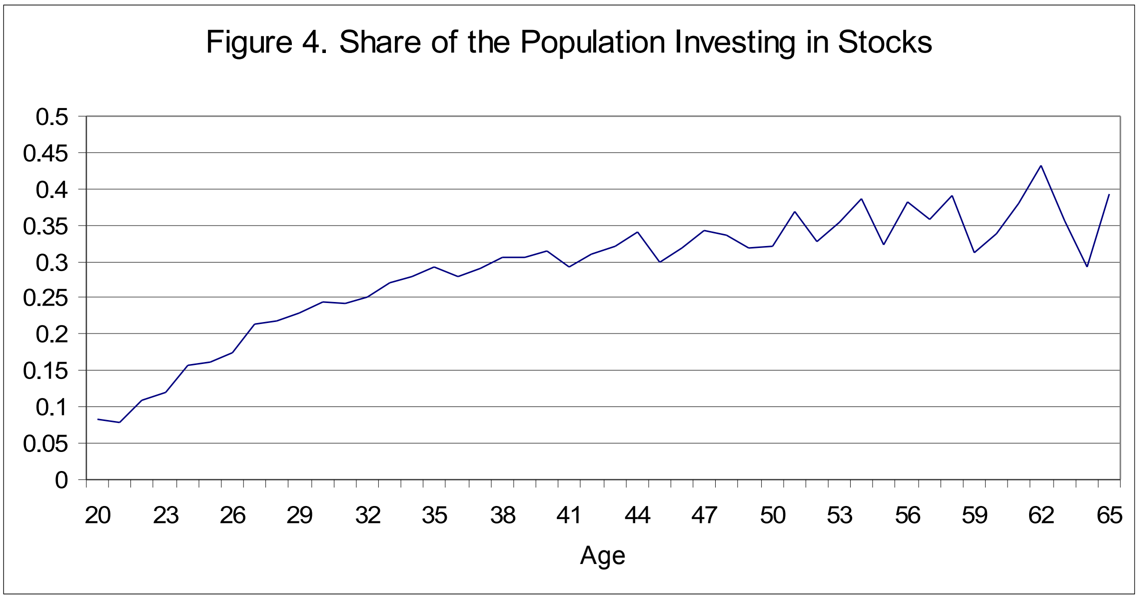


Figure 4. Share of the Population Investing in Stocks
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
20 23 26 29 32 35 38 41 44 47 50 53 56 59 62 65
Age

Figure 5. Wealth, Wage Income and Consumption for Constrained Agent

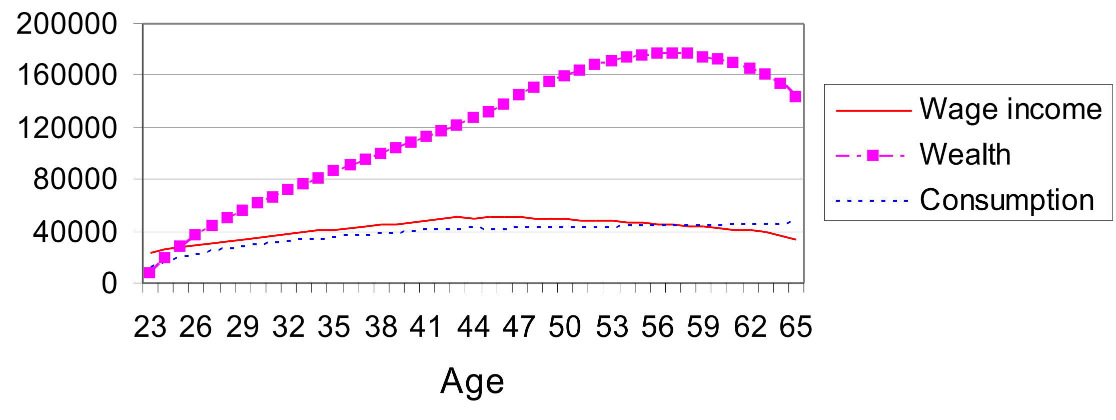


Figure 6. Wealth, Wage Income and Consumption for Unconstrained Agent

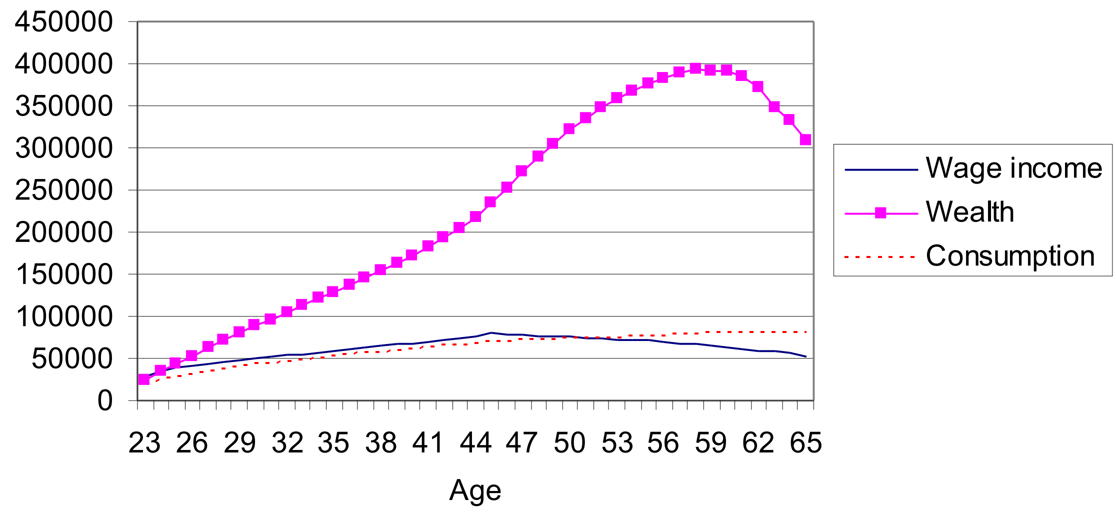


Figure 7. Share of Risky Asset in the Portfolio

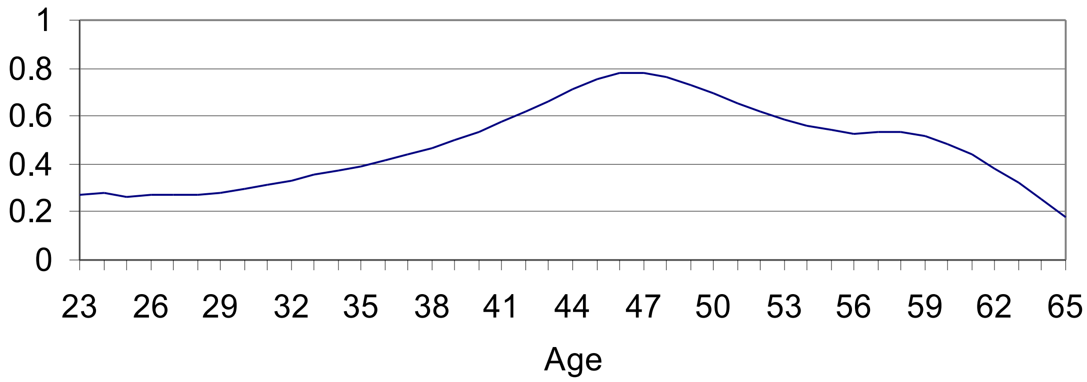
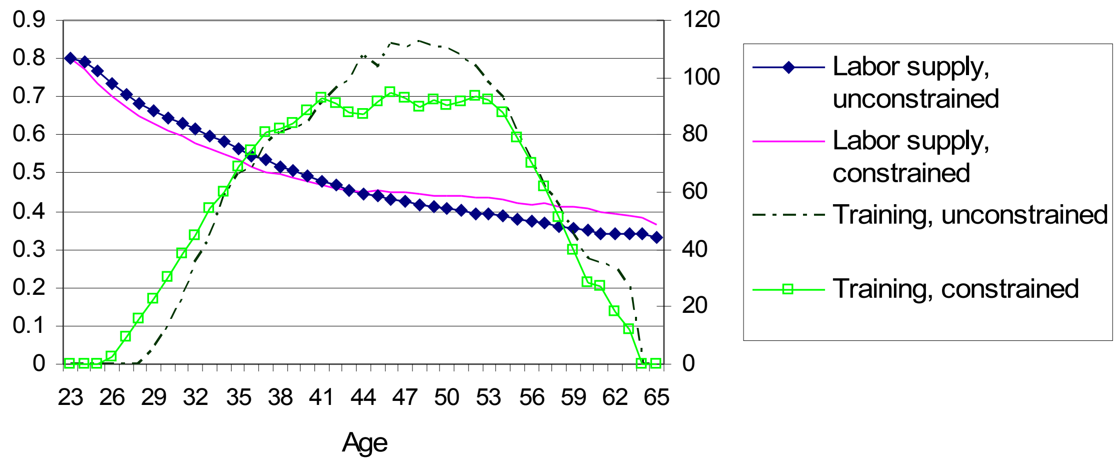


Figure 8. Labor Supply and Training



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